Fair shares? by François Macé

1) **Mrs Jenkins' shares:**

Without any calculations, I think the first child ended up with the most money.

	Lump sum	1/6 of the money remaining	Total money received by the child
First child	£1	(£25 - £1)/6 = £4	£5
Second child	£2	(£25 - £5 - £2)/6 = £3	£5
Third child	£3	(£25 - 2*£5- £3)/6 = £2	£5
Fourth child	£4	(£25 - 3*£5 - £4)/6 = £1	£5
Fifth child	£5	(£25 - 4*£5 - £5)/6 = £0	£5

I was wrong because each child received the same amount of money (£5).

2) **Mrs Hobson' shares:**

Let n be the number of Mrs Hobson's children and x the amount of money she shared with her family.
All the children received the same amount of money. Therefore each child received: x/n.

Her first child will receive:
1 + 1/5 of the money remaining = 1 + (x - 1)/5
= (5 + x -1)/5
= (x + 4)/5
Because each child received x/n, then
(x + 4)/5 = x/n
(x + 4)* n = 5x
nx + 4n = 5x
4n = 5x - nx
4n = (5 - n)*x
x= 4n/(5 - n) and 1< n< 5 because you cannot divide by 0 so n ≠ 5 and Mrs Hobson has at least 2 children.

If n = 2, x = 8/3 = £2.66 and each child will receive (2.66 + 4)/5 = £1.13.
This is not possible because the second child must receive £2 plus 1/5 of the money remaining, which is more than £1.13!

If n = 3, x = £6 and each child will receive (6 + 4)/5 = £2.
But it is not possible because the third child must receive £3 plus 1/5 of the money remaining, which is more than £2!

If n = 4, x= £16 and each child will receive (16+4)/5 = £4. Let's check:

	Lump sum	1/5 of the money remaining	Total money received by the child
First child	£1	(£16 - £1)/5= £3	£4
Second child	£2	(£16 - £4 - £2)/5 = £2	£4
Third child	£3	(£16 - 2*£4 - £3)/5 = £1	£4
Fourth child	£4	(£16 - 3*£4 - £4)/5 = £0	£4

This solution works. So Mrs Hobson shared £16 among her 4 children, who each received £4.

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3) Let x be the amount of money the mother shared with her 8 children and 1/y the fraction of the remainder.
Her 8 children received the same amount of money. Therefore each child received: x/8.

Her first child will receive:

$1 + 1/y$ of the money remaining $= 1 + (x - 1)/y$
$= (y + x - 1)/y$

Because each child received x/n, then

$(y + x - 1)/y = x/8$

$(y + x - 1)*8 = xy$

$8y + 8x - 8 = xy$

$8x - 8 = xy - 8y$

$8x - 8 = (x - 8)y$

$Y = (8x - 8)/(x - 8)$

x is a multiple of 8 and is greater than or equal to 64, because each child must receive the same amount of money and the eighth child must receive at least £8.

y is greater than 8 (the number of children) because the eight children received a lump sum in addition to the fraction of the money remaining.

If x = 64, y = (8*64 - 8)/(64 - 8)= 504/56 = 9. Let's check:

	Lump sum	1/9 of the money remaining	Total money received by the child
First child	£1	(£64 - £1)/9= £7	£8
Second child	£2	(£64 - £8 - £2)/9 = £6	£8
Third child	£3	(£64 - 2*£8- £3)/9 = £5	£8
Fourth child	£4	(£64 - 3*£8 - £4)/9 = £4	£8
Fifth child	£5	(£64 - 4*£8 - £5)/9 = £3	£8
Sixth child	£6	(£64 - 5*£8 - £6)/9 = £2	£8
Seventh child	£7	(£64 - 6*£8 - £7)/9 = £1	£8
Eight child	£8	(£64 - 7*£8 - £8)/9 = £0	£8

This solution works. So the mother shared £64 among her 8 children, who each received £8.

4) For any number of children n, money can be shared out in this way if:

- the amount of money x is equal to the square of the number of children n

$$x = n^2$$

- and the fraction 1/y is equal to 1/(n+1), which means that y= n+1.

Therefore each child receive $£(x/n) = £(n^2/n) = £n$.

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5) <u>Hypothesis:</u> A mother can share £n^2 among her n children by giving a lump sum plus a fraction 1/(n+1) of the remainder. Each children will receive £n. Let's check:

	Lump sum £	1/(n+1)of the money remaining £	Total money received by the child £
First child	1	$(n^2 - 1)/(n+1)$	n
Second child	2	$(n^2 - n - 2)/(n+1)$	n
Third child	3	$(n^2 - 2n - 3)/(n+1)$	n
....	....	....	....
(n-2)th child	n-2	$(n^2 - (n - 3)n - (n - 2))/(n+1)$	n
(n-1)th child	n-1	$(n^2 - (n - 2)n - (n - 1))/(n+1)$	n
nth child	n	$(n^2 - (n - 1)n - n)/(n+1)$	n

The total of the lump sum is:

$1 + 2 + 3 + ... + (n-2) +(n- 1) + n = n(n+1)/2$

The total of 1/(n+1) of the money remaining is:

$((n^2 - 1) + (n^2 - n - 2) + (n^2 -2n - 3) + ... (n^2 - (n - 3)n -(n - 2)) + (n^2 - (n - 2)n -(n - 1))+ (n^2- (n - 1)n - n))/(n+1)$

$= (n* n^2 -(1 + 2 + 3 +.... + (n - 2) + (n-1) + n) - (n +2n + ... + (n-3)n+(n-2)n+ (n-1)n))/ (n+1)$

$= (n^3 - n(n+1)/2 - n(1 + 2 + 3 +.... + (n- 3) + (n-2) +(n-1))/ (n+1)$

$= (n^3- n(n+1)/2 - n((n-1)n)/2))/ (n+1)$

$= (n^3 - n(n+1)/2 – n^2(n-1)/2)/(n+1)$

$= (2n^3 - n^2-n – n^3+ n^2))/2(n+1)$

$= (n^3-n)/2(n+1)$

$= n(n^2-1)/2(n+1)$

$= n(n-1)(n+1)/2(n+1)$

$= n(n-1)/2$

The total money received by all the children is: n^2

So we need to check that :

total of the lump sum + total of 1/(n+1) of the money remaining = total money received by all the children

$n(n+1)/2 + n(n-1)/2 = n^2$

$½(n^2+n +n^2-n) = n^2$

$½(2n^2) = n^2$

$n^2 = n^2$

So my hypothesis was right. A mother can share £n^2 among her n children by giving a lump sum plus a fraction 1/(n+1) of the remainder. Each children will receive £n.