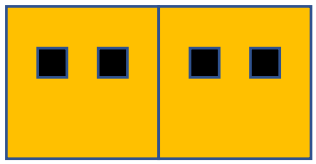
A City of Towers

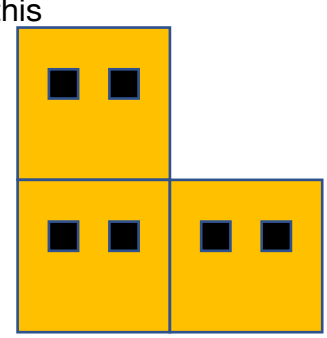
Once upon a time there was an island in the middle of a great blue sea. It was a very beautiful place with many woods and gardens and fields. Because it was such a lovely place, a great many people wanted to live there.

The Queen of the island was worried. If so many people came to the Island, then they would need somewhere to live. People might start chopping down trees in the woods and building houses in the gardens and fields. So the Queen asked a Sage, that is a very wise person, what should be done. The Sage said that when more houses were needed people should not build in all the woods, gardens and fields but that all houses should have just two rooms on the ground floor. Those houses were for two people.



There would be just one room for everyone in the house. So when children were born or other people moved into the house, one room could be added for each person. These rooms had to be on top of the two rooms that were already there.
A house for three people could look like

this



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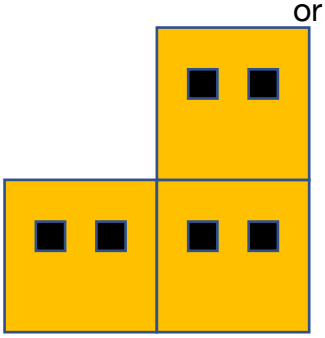

So as the city grew bigger with more people living there, the houses grew taller and taller. It soon came to be known as the City of Towers. And still it was surrounded by those beautiful woods and gardens and fields.

Now in the City of Towers there were some families of four people. In how many different ways do you think they could build their houses?
In how many different ways could a family of five build their house?

What do you notice?

Can you predict how many ways there will be for a family of seven to build their house?
Will your noticing always be true? Can you create an argument that would convince mathematicians?