

$$\sqrt{x} + \frac{1}{\sqrt{x}} < 4$$

Since  $\sqrt{x} > 0 \forall x \in \mathbb{R}^+$ , we can multiply everything by  $\sqrt{x}$ .

$$x + 1 < 4\sqrt{x}$$

$$\therefore x - 4\sqrt{x} + 1 < 0$$

We can now set  $y = \sqrt{x}$  :

$$y^2 - 4y + 1 < 0$$

Given that the coefficient of the squared term is positive, the trinomial will be negative in the open interval between its roots.

$$\text{let } y^2 - 4y + 1 = 0$$

$$\therefore y = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$$

$$\therefore 2 - \sqrt{3} < y < 2 + \sqrt{3}$$

$$\therefore 2 - \sqrt{3} < \sqrt{x} < 2 + \sqrt{3}$$

Since everything is positive we can square all parts to obtain the range of x values for which the first inequality is satisfied:

$$(2 - \sqrt{3})^2 < x < (2 + \sqrt{3})^2$$