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Solution to the 'In Between' Problem

Solve $\sqrt{x} + (1/\sqrt{x}) < 4$

Since we are only concerned with the positive square root of x with this inequality, we can simply multiply both sides of the inequality by $\sqrt{x}$ **:**

$$\sqrt{x} + (1/\sqrt{x}) < 4$$

$$\rightarrow x + 1 < 4\sqrt{x}$$

$$\rightarrow x - 4\sqrt{x} + 1 < 0$$

We know from index rules that $\sqrt[n]{x} = x^{1/n}$ *and* $(x^a)^b = x^{ab}$ *so we can write the inequality above in the following form:*

$$(x^{1/2})^2 - 4x^{1/2} + 1 < 0$$

Now let $f(x) = (x^{1/2})^2 - 4x^{1/2} + 1$
We can see the f(x) is in the form of a quadratic so we can find the critical values by setting f(x) = 0 . Hence to find the solutions to f(x) = 0, we can use the quadratic formula:

$$x^{1/2} = (-b \pm \sqrt{b^2-4ac})/2a \quad \text{where } a=1,\ b=-4 \text{ and } c=1$$

$$\rightarrow x^{1/2} = (4 \pm \sqrt{(-4)^2-4(1)(1)})/2$$

$$= (4 \pm 2\sqrt{3})/2$$

$$= 2 \pm \sqrt{3}$$

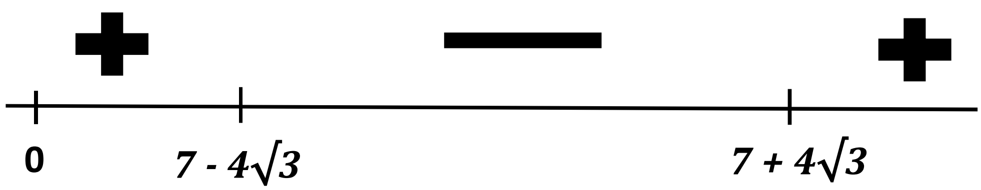
$$\therefore x^{1/2} = 2+\sqrt{3} \quad or \quad 2 - \sqrt{3}$$

$$\rightarrow x = (2+\sqrt{3})^2 \qquad or \qquad x = (2-\sqrt{3})^2$$

$$= 7 + 4\sqrt{3} \qquad\qquad = 7 - 4\sqrt{3}$$

We now know that the solutions to f(x)=0 are x= 7 ± 4$\sqrt{3}$. However, to satisfy the inequality x would have to take a range of values so we need to know the regions for the graph of f(x) which are positive and negative. To do this, we can substitute values of x into f(x) between the critical values and either side of them:

f(4) = -3 *f(0) = 1* *f(16) = 1*



As the original inequality is ***f(x) < 0*** *we are only interested in the negative region of the function so therefore the range of values of x that satisfy the inequality is:*

$$\mathbf{7-4\sqrt{3} < x < 7+4\sqrt{3}}$$