$\sqrt{x} + \frac{1}{\sqrt{x}} < 4$

$\sqrt{x} - 4 + \frac{1}{\sqrt{x}} < 0 \rightarrow$ rearrange to be less than 0

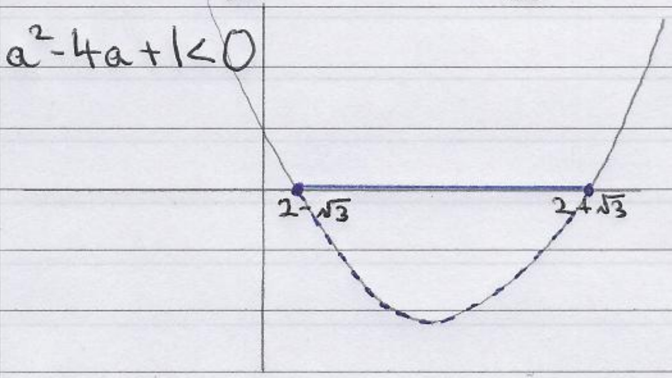
$\rightarrow$ given that $\sqrt{x}$ is positive, we can

$x - 4\sqrt{x} + 1 < 0$ multiply both sides by $\sqrt{x}$ and the '<' sign won't change

letting $a = \sqrt{x}$, $a^2 - 4a + 1 < 0 \rightarrow$ make it a quadratic

$$a = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

$a^2 - 4a + 1 < 0$



$\Rightarrow 2 - \sqrt{3} < a < 2 + \sqrt{3}$

$\Rightarrow 2 - \sqrt{3} < \sqrt{x} < 2 + \sqrt{3}$

$2 - \sqrt{3}$ and $2 + \sqrt{3}$ are positive, and $f(x) = \sqrt{x}$ is a non-negative, increasing function. Squaring them keeps them positive, so doesn't change the inequality signs

squaring $2 - \sqrt{3} < \sqrt{x} < 2 + \sqrt{3}$:

$(2 - \sqrt{3})^2 < x < (2 + \sqrt{3})^2$

$4 - 4\sqrt{3} + 3 < x < 4 + 4\sqrt{3} + 3$

$\Rightarrow \quad 7 - 4\sqrt{3} < x < 7 + 4\sqrt{3}$