



Alison's Method

There are 3 possible places where a line can start:

- at a vertex
- at the middle of an edge
- in the centre of a face

A cube has 8 vertices, 12 edges and 6 faces.

From a **vertex** there are 7 other vertices that you can join to in order to make a winning line. $7 \times 8 = 56$ lines, but this counts each line from both ends, so there are 28 'vertex winning lines'.

From the **middle of an edge** there are 3 other middles-of-edges that you can join to in order to make a winning line. $3 \times 12 = 36$ lines, but this counts each line from both ends so there are 18 'middle of edge' winning lines.

From the **centre of each face** there is one winning line, joining to the opposite face, so there are 3 'centre of face' winning lines.

So in total, there are $28 + 18 + 3 = 49$ winning lines.

James' Method

Winning lines can either be:

- Diagonal
- Not diagonal

Considering the **non-diagonal winning lines** first:

There are 9 from front to back.

There are 9 from left to right.

There are 9 from top to bottom.

Considering the **diagonal winning lines**:

On each layer there are 2 diagonal winning lines so:

There are 6 from front to back.

There are 6 from top to bottom.

There are 6 from left to right.

There are 4 lines from a vertex to a diagonally opposite vertex.

In total, there are $27 + 18 + 4 = 49$ winning lines.



Caroline's Method

All winning lines must pass either:

- along an edge of the cube
- through the middle of a face
- through the centre of the cube

There are 12 edges on a cube so there are 12 winning lines **along edges**.

There are 6 faces on a cube, and 4 winning lines that pass through the middle of each face, so there are 24 winning lines **through the middle of faces**.

Finally we need to consider the winning lines that go **through the centre cube**:

vertex to opposite vertex: 4

middle of edge to middle of opposite edge: 6

middle of face to middle of opposite face: 3

In total, there are $12 + 24 + 4 + 6 + 3 = 49$ winning lines.

Grae's Method

The winning lines may be counted by looking at lines:

- in each horizontal plane
- in each vertical plane from left to right
- in each vertical plane from front to back
- in the diagonal planes

On a plane there are 8 winning lines.

In the cube, there are 3 **horizontal planes**, so $8 \times 3 = 24$ winning lines.

There are also 3 **vertical planes going from left to right**, but now with only 5 **new** winning lines per plane, as the 3 horizontal lines have already been counted. So $5 \times 3 = 15$ winning lines.

On the 3 **vertical planes going from front to back**, we now only have 2 **new** (diagonal) winning lines per plane. So $2 \times 3 = 6$ winning lines.

Finally, there are also **diagonal planes** to consider. There are 4 winning lines going from corner to diagonally opposite corner.

In total, there are $24 + 15 + 6 + 4 = 49$ winning lines.