Extension for **Buckets of Thinking**

The problem can be opened out to much greater things that will also appeal to older, higher-attaining or simply motivated pupils.

This can be done by asking some simple questions of the pupils:

"What other quantities of liquid in the buckets can you come up with? Can you provide two or three statements that will challenge someone else to know what there is in each bucket?" (keeping the rule that there's a different amount in each as whole litres and 5 litres is the maximum).

A response by the pupils, to this, might go as follows:
A new set of quantities could be:-
The Red has 3 litres, the Blue 4 litres and the Yellow 5 litres

Suggested statements
1. "You cannot fit Red or Blue into the Yellow." [because Y will then be greater than 5]

So on hearing that statement the challenged pupil could try:
Y as 1 [No because 2, 3, or 4 could go with that]
Y as 2 [Yes then R & B could be 4 & 5 in either order]
Y as 3 [Yes again R & B could be 4 & 5 in either order]
Y as 4 [Yes now R & B could be any two out of 2, 3 or 5]
Y as 5 [Yes now R & B could be any two out of 1, 2, 3 or 4]

The challenged pupil is left with twenty-two possibilities. So the questioner would have to think of another good statement.

Maybe:
2. "Red, Blue and Yellow go up in equal steps."

So on hearing that statement the challenged pupil could try:
R=1, B=2, Y=3 [No because it doesn't match statement 1]
R=2, B=3, Y=4 [Yes 2+4 and 3+4 are greater than 5]
R=3, B=4, Y=5 [Yes 3+5 and 4+5 are greater than 5]
The challenged pupil now has two possibilities.

Then the challenger could make a third statement, like:

3. “Half of B can be put into R and that would make R the same as Y.”

This would allow the challenged pupil to find a unique solution.

What a lot of really good thinking has had to happen! Activities opened out like this give both opportunities and encouragement for such thinking to take place.

To extend further, more questions could be asked as follows:

A. Keeping just the rules that 5 litres is the maximum and that all three contain different amounts, what possibilities are there?

B. Repeat A. above but what if there were FOUR buckets?

C. Repeat A. above but change the maximum number of litres.

D. What about the many possibilities when the numbers of buckets and the numbers of litres are both changed?

This could lead to a table like the one below which shows the total number of possible arrangements for any number of buckets of any capacity. For example, if there are two buckets which can each hold 4 litres, there are six different possibilities for the amount of liquid they can contain (if we assume they must hold at least one litre and they cannot hold the same amount):
1 2, 1 3, 1 4
2 3, 2 4
3 4

				L	I	T	R	E	S	
		2	3	4	5	6	7	8	9	10
	2	1	3	6	10	15	21	28	36	45
B	3		1	4	10	20	35	56	84	120
U	4			1	5	15	35	70	126	210
C	5				1	6	21	56	126	252
K	6					1	7	28	84	210
E	7						1	8	36	120
T	8							1	9	45
S	9								1	10
	10									1

This can then be used to introduce more ideas about combinations and Pascal's Triangle:

								1								
							1		1							
						1		2		1						
					1		3		3		1					
				1		4		6		4		1				
			1		5		10		10		5		1			
		1		6		15		20		15		6		1		
	1		7		21		35		35		21		7		1	
1		8		28		56		70		56		28		8		1

The questions that have been asked to open out this simple activity are often transferable to other activities that seem so simple at first. Have a go!