Always Perfect

Part 2(a)
Consider what happens when 1 is added to the product of four consecutive numbers, for example $2\times3\times4\times5+1=121$.

Try this for some other sets of four consecutive numbers.
What do you notice? Is this always true? Can you prove your conjecture?

Here are some ideas to help you prove the conjecture.

Charlie noticed a relationship between the product of the first and last of the four consecutive numbers, and the solution:

$$2\times3\times4\times5+1=121=(10+1)^2 \qquad 3\times4\times5\times6+1=361=(18+1)^2$$

He then used this to help him write the general rule algebraically and check that it worked.

Claire expressed what she was doing algebraically:

$$n(n+1)(n+2)(n+3) = n^4 + 6n^3 + 11n^2 + 6n + 1$$

and then she tried to factorise this to find a squared expression.

Part 2(b)
Now take four consecutive even numbers (or four consecutive odd numbers), multiply them together and add 16 (so for example $2\times4\times6\times8+16=400$).

What do you notice? Can you make and prove a conjecture?

Part 2(c)
Now take four numbers which differ by 3, multiply them together and add 81 (so for example $2\times5\times8\times11+81=961$).

What do you notice? Can you make and prove a conjecture?

Part 2(d)
Looking at your results for parts (a), (b) and (c) can you predict what you will have to add to the product of four numbers that differ by 4,5,6,...,k in order to get a square number?

Can you write down a general statement? Can you prove this statement?

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