Can you find a number with the property that when it is divided by each of the numbers 2 to 10, the remainder is always one less than the number it is has been divided by?

Can you find the smallest number that satisfies this condition?

I started by looking at the divisibility properties from the numbers 2 to 10.

<u>**Divisor 10**</u>: The remainder should be in the units place. In this case, the number should end with the number 9.

<u>**Divisor 2**</u>: Since the number ends with 9, which make the number odd, there is a remainder of 1.

<u>**Divisor 5**</u>: Since 9 is in the units place, and 9 is 4 more than 5, there is a remainder of 4.

<u>**Divisor 3**</u>: To get a remainder of 2, the sum of the number's digits should be 2, 5 or 8.

E.g. 11÷3 (the quotient is 3 remainder 2) the sum of 11's digits is 2.

14÷3 (the quotient is 4 remainder 2) the sum of 14's digits is 5.

17÷3 (the quotient is 5 remainder 2) the sum of 17's digits is 8.

<u>**Divisor 6**</u>: For number six, you need to do the same thing you did for number 3.

E.g. 11÷6 (the quotient is 1 remainder 5) the sum of 11's digits is 2.

17÷6 (the quotient is 2 remainder 5) the sum of 17's digits is 8.

23÷6 (the quotient is 3 remainder 5) the sum of 23's digits is 5.

<u>**Divisor 9**</u>: For number 9, you need to add the number's digits to get the remainder. The remainder is the same number as the sum of the number's digits.

E.g. 53÷9 (the quotient is 5 remainder 8) the sum of the digits is 8.

22÷9 (the quotient is 2 remainder 4) the sum of the digits is 4.

<u>**Divisor 4**</u>: Since our number ends with 9, to get a remainder of 3, we need our number to end with 19, 39, 59, 79 or 99.

<u>**Divisor 8**</u>: If you want a remainder of seven, and your number ends with 19, 59 or 99, then the hundredth place should be 1, 3, 5, 7 or 9. This is because in the second part of 8's cycle, 19, 59 and 99, when divided by 8, will have a remainder of 7. For 39 and 79, the hundredths place should be 2, 4, 6 or 8. Take away 7 from the number. If the last 3 digits are divisible by 8, you have a remainder of 7. Based on this rule we can eliminate numbers.

119, 159, 199, 239, 279, 319, 359, 399, 439, 479, 519, 559, 599, 639, 679, 719, 759, 799, 839, 879, 919, 959 and 999 give a remainder of 7 when divided by 8

***The number cannot be a 2-digit number because*:**

19, 79: These numbers don't work because, when divided by 3, they do not give a remainder of 2.

39, 99: These numbers don't work because they are divisible by 3.

59 : This number doesn't work because, when divided by 7, it gives a remainder of 3.

Now, we can start looking for 3 digit numbers.

We know that our number should end with 19, 39, 59, 79 and 99, and that its digits should add up to 8.

The numbers are now :

179, 539 : These numbers won't work because, when divided by 8, they do not give a remainder of 7.

359, 719, 899 : These numbers won't work because, when divided by 7, they do not give a remainder of 6.

629, 269, 449, 809 : These numbers won't work because, when divided by 4, it gives a remainder of 1. Numbers that end with 09, 29, 49, 69 and 89 should be ignored.

Therefore, the number is not a three digit number.

The number can now be :

1169, 1619, 6119, 1259, 1529, 2159, 2519, 5129, 5219, 2249, 2429, 4229, 1349, 1439, 3149, 3419, 4139, 4319, 3239, 3329, 2339, 8009, 7109, 7019, 1079, 1709, 6209, 6029, 2069, 2609, 5309, 5039, 3059, 3509, 4409, 4049

1619, 2339, 1259, 5219, 3419, 4139, 7019, 3059 : These numbers won't work because, when divided by 8, they do not give a remainder of 7.

3239, 4319, 6119, 1079, 2159, 1439 : These numbers won't work because, when divided by 7, they do not give a remainder of 6.

2519, 5039 : These numbers do work!

1169, 1529, 5129, 1349, 3149, 2249, 2429, 4229, 3329, 8009, 7109,1709, 6209, 6029, 2069, 2609, 5309, 3509, 4409, 4049 : These numbers won't work because, when divided by 4, it gives a remainder of 1. Numbers that end with 09, 29, 49, 69 and 89 should be ignored.

The answers are **5039** and **2519**. They satisfy all the conditions.