Tetra ~~perf~~ perp

In order to prove a statemat in the form '~~If~~ Show A if and only if B' we must prove 2 things:

1. If A is true then B is true
2. If B is true then A is true

Firstly we note that ABCD is irregular and we are given no lengths / angles so we will most likely need to be dealing with vectors so we may represent the points A, B, C, and D as position vectors from an origin O:

$\vec{OA} = a$, $\vec{OB} = b$, $\vec{OC} = c$, $\vec{OD} = d$

we could firstly attempt to prove statemat 1. For this we need lengths which we know are also magnitudes so we might try to find vectors and thus ~~the~~ lengths of: (AB)*, (CD), (AC), and (BD) as these are in the statement we want to prove.

$\vec{AB} = \vec{AO} + \vec{OB} = b - a$; $(AB) = |\vec{AB}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\therefore (AB)^2 = 2$

* (AB) means length AB.

$\vec{CD} = \vec{CO} + \vec{OD} = d - c$; $(CD) = |\vec{CD}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\therefore (CD)^2 = 2$

$\vec{AC} = \vec{AO} + \vec{OC} = c - a$; $(AC) = |\vec{AC}| = \sqrt{1 + (-1)^2} = \sqrt{2}$

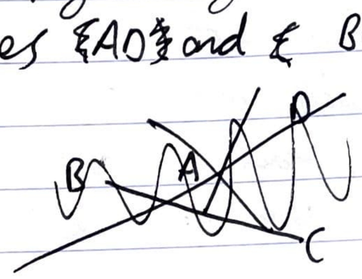
$\therefore (AC)^2 = 2$

$\vec{BD} = \vec{BO} + \vec{OD} = d - b$; $(BD) = |\vec{BD}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

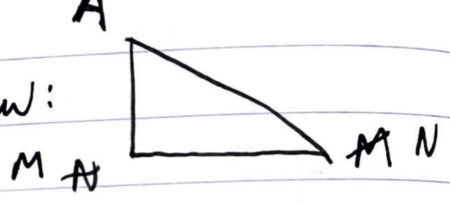
$\therefore (BD)^2 = 2$

Substituting these values into the formula $AB^2 + CD^2 = AC^2 + BD^2$ gives
$2 + 2 = 2 + 2$ which is true so we have proved that this formula is always true which means that 'if A then B' has been proven.

Now we must prove statement 2. We note that two lines are perpendicular if the angle between them is a right angle. So we could draw a line connecting a lines {AD} and {BC}:



Side view:



where N is the midpoint of BC and M is the midpoint of AD. We know we can prove there is a right angle at ANM if pythagoras is true with hypotenuse AN. So we need to find lengths: (AN), (AM), and (NM) so using vectors again:

$\vec{AN} = \vec{AB} + \vec{BN} = b - a + \frac{1}{2}\vec{BC} = b - a + \frac{c - b}{2} = \frac{b}{2} + \frac{c}{2} - a$; $(AN) = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2 + (-1)^2}$

$\vec{AM} = \frac{1}{2}\vec{AD} = \frac{d - a}{2}$; $(AM) = |\vec{AM}| = \sqrt{(\frac{1}{2})^2 + (-\frac{1}{2})^2} = \sqrt{1/2}$

$\vec{NM} = \vec{NA} + \vec{AM} = -\frac{b}{2} - \frac{c}{2} + a + \frac{d - a}{2} = \frac{d}{2} + \frac{a}{2} - \frac{b}{2} - \frac{c}{2}$

$(NM) = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2 + (-\frac{1}{2})^2 + (-\frac{1}{2})^2} = \sqrt{1} = 1$

So $(NM)^2 + (AM)^2 = (AN)^2$ ✓

$1^2 + \sqrt{\tfrac{1}{2}}^2 = \sqrt{\tfrac{3}{2}}^2$ or $1 + \tfrac{1}{2} = \tfrac{3}{2}$

So this means if B is true (always) then A is true.