Another nice problem from Nrich: Really numbers, from 1st epsilons

If x, y and z are real numbers such that:

$$x+y+z=5 \quad \text{and} \quad xy+yz+zx=3,$$

what is the largest value that any one of these numbers can have?

I had some trouble following the solutions using maxima /minima of curves.

However I propose the following:

$$(x+y+z)^2 = 25$$

$$x^2+y^2+z^2+2(xy+yz+xz) = x^2+y^2+z^2+6$$

Therefore $$x^2+y^2+z^2 = 19$$

Possible solutions: $x=\pm 3, \quad y=\pm 3, \quad z=\pm 1$

The only combination satisfying the required conditions is:

$$x=3, \quad y=3, \quad z=-1$$