How Old Am I?

First part:

a=age

$(a-15)^2=a+15$

$a^2-30a+225=a+15$

$a^2-31a+210=0$

$(a-21)(a-10)=0$

age=21 (a=10 gives a-15=-5)

Second part:

Try the other numbers:

$(a-3)^2=a+3$

$a^2-7a+6=0$

$(a-6)(a-1)=0$ so age=6

$(a-4)^2=a+4$

$a^2-9a+12=0$ no integer solution

$(a-5)^2=a+5$

$a^2-11a+20=0$ no integer solution

$(a-6)^2=a+6$

$a^2-13a+30=0$

$(a-10)(a-3)=0$ so age is 10

So 3,6 and 15 work. It looks as if there is a connection with triangular numbers, so I tried 10:

$(a-10)^2=a+10$

$a^2-21a+90=0$

$(a-15)(a-6)=0$ so age is 15

So all the triangular numbers seem to work and the age is the triangular number greater than the one in the question.

To prove it:

Does it work for k?

$(a-k)^2=a+k$

$a^2-(2k+1)a+k^2-k=0$

$$a=\frac{2k+1\pm\sqrt{(2k+1)^2-4k^2+4k}}{2}$$

$$=\frac{2k+1\pm\sqrt{8k+1}}{2}$$

This has integer solutions when 8k+1 is a square number.

This is true if k is a triangular number.

(see diagram when k=10)