

$$\frac{xy}{x+y} = \frac{1}{2}, \quad \frac{yz}{y+z} = \frac{1}{3}, \quad \frac{xz}{x+z} = \frac{1}{7}, \quad \text{solve for } x, y \text{ and } z.$$

$$\begin{aligned} 1. \quad \frac{xy}{x+y} &= \frac{1}{2} \\ 2 &= \frac{x+y}{xy} \\ 2 &= \frac{x}{xy} + \frac{y}{xy} \\ 2 &= \frac{1}{y} + \frac{1}{x} \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} 2. \quad \frac{yz}{y+z} &= \frac{1}{3} \\ 3 &= \frac{y+z}{yz} \\ 3 &= \frac{y}{yz} + \frac{z}{yz} \\ 3 &= \frac{1}{z} + \frac{1}{y} \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} 3. \quad \frac{xz}{x+z} &= \frac{1}{7} \\ 7 &= \frac{x+z}{xz} \\ 7 &= \frac{x}{xz} + \frac{z}{xz} \\ 7 &= \frac{1}{z} + \frac{1}{x} \quad \text{--- (3)} \end{aligned}$$

$$\begin{aligned} 4. \quad \text{(1) - (2) + (3)} \\ 2-3+7 &= \left(\frac{1}{x} + \frac{1}{y}\right) - \left(\frac{1}{y} + \frac{1}{z}\right) + \left(\frac{1}{x} + \frac{1}{z}\right) \\ 6 &= \frac{1}{x} + \frac{1}{x} + \frac{1}{y} + \frac{1}{z} - \frac{1}{y} - \frac{1}{z} \\ 6 &= \frac{2}{x} \\ x &= \frac{1}{3} \end{aligned}$$

$$\begin{aligned} 5. \quad \text{From (1), substitute } x = \frac{1}{3} \\ 2 &= \frac{1}{\frac{1}{3}} + \frac{1}{y} \\ 2 &= 3 + \frac{1}{y} \\ -1 &= \frac{1}{y} \\ y &= -1 \end{aligned}$$

$$\begin{aligned} 6. \quad \text{From (2), substitute } x = \frac{1}{3} \\ 3 &= \frac{1}{\frac{1}{3}} + \frac{1}{z} \\ 3 &= 3 + \frac{1}{z} \\ 4 &= \frac{1}{z} \\ z &= \frac{1}{4} \end{aligned}$$

Therefore, $x = \frac{1}{3}$, $y = -1$, $z = \frac{1}{4}$