$\frac{xy}{x+y} = \frac{1}{2}$, $\frac{yz}{y+z} = \frac{1}{3}$, $\frac{xz}{x+z} = \frac{1}{7}$, solve for x, y and z

1. $\frac{xy}{x+y} = \frac{1}{2}$

$2 = \frac{x+y}{xy}$

$2 = \frac{x}{xy} + \frac{y}{xy}$

$2 = \frac{1}{x} + \frac{1}{y}$ —— ①

2. $\frac{yz}{y+z} = \frac{1}{3}$

$3 = \frac{y+z}{yz}$

$3 = \frac{y}{yz} + \frac{z}{yz}$

$3 = \frac{1}{y} + \frac{1}{z}$ —— ②

3. $\frac{xz}{x+z} = \frac{1}{7}$

$7 = \frac{x+z}{xz}$

$7 = \frac{x}{xz} + \frac{z}{xz}$

$7 = \frac{1}{x} + \frac{1}{z}$ —— ③

4. ① - ② + ③

$2 - 3 + 7 = \left(\frac{1}{x} + \frac{1}{y}\right) - \left(\frac{1}{y} + \frac{1}{z}\right) + \left(\frac{1}{x} + \frac{1}{z}\right)$

$6 = \frac{1}{x} + \frac{1}{x} + \frac{1}{y} + \frac{1}{z} - \frac{1}{y} - \frac{1}{z}$

$6 = \frac{2}{x}$

$x = \frac{1}{3}$

5. From ① , substitute $x = \frac{1}{3}$

$2 = \frac{1}{\frac{1}{3}} + \frac{1}{y}$

$2 = 3 + \frac{1}{y}$

$-1 = \frac{1}{y}$

$y = -1$

6. From ② , substitute $x = \frac{1}{3}$

$7 = \frac{1}{\frac{1}{3}} + \frac{1}{z}$

$7 = 3 + \frac{1}{z}$

$4 = \frac{1}{z}$

$z = \frac{1}{4}$

Therefore , $x = \frac{1}{3}$, $y = -1$, $z = \frac{1}{4}$