Special 24 Extension Ideas

Initially this is obviously a closed question. We're given that this special thing happens with 24 and the pupils work in different ways to get an answer.

BUT there are ways of opening this out by exploring the initial ideas and changing rules to investigate further.

Assuming that the answer has been found, we can:

A/ Ask the pupils to look at what happens generally when you perform the same operations [minus 1, double, add 1] on the square numbers [number 25 being the one in the question]. As is often the case it is worthwhile looking at the digital roots of the numbers we get. [See the article, Digital Roots.]

	square	minus 1	doubled	D.R.	plus 1	sqrt
1	1	0	0	9	1	1
2	4	3	6	6	7	2.65
3	9	8	16	7	17	4.12
4	16	15	30	3	31	5.57
5	25	24	48	3	49	7
6	36	35	70	7	71	8.43
7	49	48	96	6	97	9.85
8	64	63	126	9	127	11.27
9	81	80	160	7	161	12.69
10	100	99	198	9	199	14.11
11	121	120	240	6	241	15.52
12	144	143	286	7	287	16.94
13	169	168	336	3	337	18.36
14	196	195	390	3	391	19.77
15	225	224	448	7	449	21.19
16	256	255	510	6	511	22.61
17	289	288	576	9	577	24.02
18	324	323	646	7	647	25.44
19	361	360	720	9	721	26.85
20	400	399	798	6	799	28.27
21	441	440	880	7	881	29.68
22	484	483	966	3	967	31.10
23	529	528	1056	3	1057	32.51
24	576	575	1150	7	1151	33.93
25	625	624	1248	6	1249	35.34
26	676	675	1350	9	1351	36.76
27	729	728	1456	7	1457	38.17
28	784	783	1566	9	1567	39.59
29	841	840	1680	6	1681	41

So that's the kind of table we end up with but when detective-like pupils look to find things, the digital roots certainly have something to offer.

B/ Ask the older or more experienced pupils to look again at the initial question - the idea of squaring a number, subtracting 1 and then doubling - and stopping just there

and exploring. Here are some examples of what you might get and some links that show up.

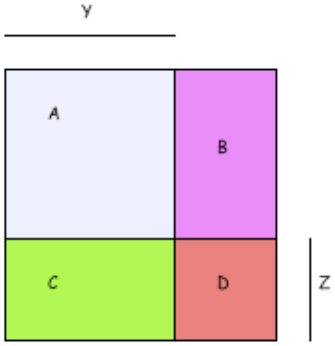
A	B	C	D
	square	minus 1	doubled
1	1	0	0
2	4	3	6
3	9	8	16
4	16	15	30
5	25	24	48
6	36	35	70
7	49	48	96
~	~	~	~
17	289	288	576
18	324	323	646
19	361	360	720
20	400	399	798
21	441	440	880
22	484	483	966
23	529	528	1056
24	576	575	1150
~	~	~	~
29	841	840	1680
30	900	899	1798
31	961	960	1920
32	1024	1023	2046
33	1089	1088	2176
34	1156	1155	2310
35	1225	1224	2448
36	1296	1295	2590
37	1369	1368	2736
38	1444	1443	2886
39	1521	1520	3040
40	1600	1599	3198
41	1681	1680	3360

Having produced a table like this the same kind of question arises that you can ask the pupils, "What things can you see?" And off you go with them on another exploration.

Key Stage 3 & 4 pupils might take this onto a spreadsheet and explore, and using the special numbers that linked in the table above, could take the whole idea further. As an example of some of the kinds of things that may show up, have a look at this [which includes the special numbers only that link up, their square root, the doubling, the amount which has to be added on (no longer minus 1 but 0, 1, 4, 9), and the final square root, and the digital roots along the way]:

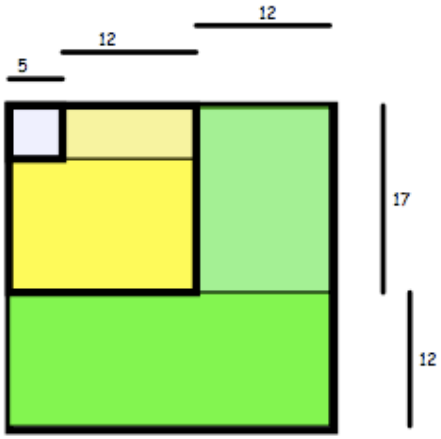
A	B	C	D	E	F	G	H	I	J	K	L
	C diff	Number	D.R.	sqrt C +1	D.R.	C x2	A sqd	G + H	D.R.	sqrt I	D.R.
0		288	9	17	8	576	0	576	9	24	6
1	552	840	3	29	2	1680	1	1681	7	41	5
2	840	1680	6	41	5	3360	4	3364	7	58	4
3	1128	2808	9	53	8	5616	9	5625	9	75	3
4	1416	4224	3	65	2	8448	16	8464	4	92	2
5	1704	5928	6	77	5	11856	25	11881	1	109	1
6	1992	7920	9	89	8	15840	36	15876	9	126	9
7	2280	10200	3	101	2	20400	49	20449	1	143	8
8	2568	12768	6	113	5	25536	64	25600	4	160	7
9	2856	15624	9	125	8	31248	81	31329	9	177	6
10	3144	18768	3	137	2	37536	100	37636	7	194	5
11	3432	22200	6	149	5	44400	121	44521	7	211	4
12	3720	25920	9	161	8	51840	144	51984	9	228	3
13	4008	29928	3	173	2	59856	169	60025	4	245	2
14	4296	34224	6	185	5	68448	196	68644	1	262	1
15	4584	38808	9	197	8	77616	225	77841	9	279	9
16	4872	43680	3	209	2	87360	256	87616	1	296	8
17	5160	48840	6	221	5	97680	289	97969	4	313	7
18	5448	54288	9	233	8	108576	324	108900	9	330	6
19	5736	60024	3	245	2	120048	361	120409	7	347	5
20	6024	66048	6	257	5	132096	400	132496	7	364	4
21	6312	72360	9	269	8	144720	441	145161	9	381	3
22	6600	78960	3	281	2	157920	484	158404	4	398	2
23	6888	85848	6	293	5	171696	529	172225	1	415	1
24	7176	93024	9	305	8	186048	576	186624	9	432	9
25	7464	100488	3	317	2	200976	625	201601	1	449	8

C/ Not forgetting this is about squares, we could do well to see the geometry involved and get the children looking at it spatially:



Small square		Y	A	B and C	Z	D		Small sq. minus 1 x2 and ?	
5^2	25	5	5x5	5x2	2	2x2	49	48+1	7^2
17^2	289	17	17x17	17x7	7	7x7	576	576+0	24^2
29^2	841	29	29x29	29x12	12	12x12	1681	1680+1	41^2
41^2	1681	41	41x41	41x17	17	17x17	3364	3360+4	58^2
53^2	2809	53	53x53	53x22	22	22x22	5625	5616+9	75^2
65^2	4225	65	65x65	65x27	27	27x27	8464	8448+16	92^2
77^2	5929	77	77x77	77x32	32	32x32	11881	11856+25	109^2
89^2	7921	89	89x89	89x37	37	37x37	15876	15840+36	126^2

or another way might be:



and saying to the pupils "Tell me what you see".

So all in all it's a matter of opening doors for the pupils, letting them look in and asking them to tell you and each other what they see.
It's an opportunity to use some skills and knowledge that have come from other areas of mathematics, for example: the four rules of number, squares and roots, digital roots, patterns, spreadsheet use, shape and space …