Developing a Classroom Culture That Supports a Problem-solving Approach to Mathematics

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What this article and its CPD activities offer

This article offers you practical ways to investigate aspects of your classroom culture. It also offers suggestions to help you develop the culture further so that students are encouraged to develop as independent mathematicians with strong problem-solving skills. This is important as we know that independent problem-solving skills are essential for students for 21st century life and work. To read more about this, have a look at the ACME report Mathematical Needs: Mathematics in the workplace and in Higher Education.

How to use this resource

You can use this article and its activities as an individual, with a colleague, in a focus group or as a whole school staff together, as you seek to offer students the highest-quality learning opportunities in mathematics.

Problem-solving skills

A problem is something you do not immediately know how to solve. There is a gap between where you are and getting started on a path to a solution. This means that your students require thinking and playing-with-the-problem time. They need to test out ideas, to make conjectures, to go up 'dead ends' and adjust their thinking in the light of what they learn from this, discuss ideas with others and be comfortable to take risks. When students are confident to behave in these ways they are then able to step into problems independently rather than immediately turning to us as teachers to ask what to do!

As teachers we can support our students to develop the skills they need to tackle problems by the classroom culture we create. It needs to be one where questioning and deep thinking are valued, mistakes are seen as useful, all students contribute and their suggestions are valued, being stuck is seen as honourable and students learn from shared discussion with the teacher, Teaching Assistant (if present) and peers.

What's happening in your classroom?

How does the cameo above compare with your classroom? We invite you to investigate this by videoing your next 'problem-solving' lesson to watch by yourself, or with a trusted colleague, and see what you notice about the key aspects detailed below.

You may like to work with this as a whole school and investigate one key aspect at a time, for example 'who does most of the talking?'. This will give you the opportunity to share good practice across the school as well as support each other in developing high-quality mathematics classrooms.

Aspects to consider	More information on each aspect
1. Who does most of the talking in whole-class parts of the lesson?	Generally, in a strong problem-solving environment the teacher needs to be doing around 30% of the talking and the students 70%. What do you notice about the balance in your classroom? What type of things are you saying when you are talking? Explaining how to do something? Asking questions?
2. What questions do I ask?	Do you ask closed questions such as, 'can you see

	how the system works?' or open questions such as, 'what system can you see emerging in this problem?'.
3. Who answers the questions?	Is it the mostly the same students? Is it the more articulate ones? Is it more often boys or girls?
4. How well do I listen to the students' answers and seek to understand what they are saying?	Do I respond by telling the whole class what I think a particular student said without checking with them? Do I slightly adjust what they said to make better sense or fit a 'better/right answer'? Do I ask the student a 'clarification' question, such as 'can I just check what I think you said was ...'?
5. What do I do with the students' answers?	Do I praise them for a fabulous answer? Do I simply evaluate their answers with comments such as 'Good', 'Well done', 'Right', 'OK', 'No', 'Think again'? Do I carry on with the next thing I was going to say? Do I ask other students to comment on what was said? Do I ask another follow-up question such as 'are you sure?' or 'how do you know that?'?
6. How do I facilitate the learning?	Do I explain how it needs to be done and make sure they understand it as fully as possible before working on their own? Do I give them key pointers/hints/clues to help them? Do I pull out the learning from the students' thinking and use that to develop the journey of the lesson?
7. How confident are the students to take a risk, to try out ideas, to make mistakes?	What evidence is there of the students taking a risk in what they offer to the discussion or ideas that they try out? What evidence is there that the students are trying out their ideas rather than replicating mine? When is it helpful for them to replicate mine? What do I do when a student makes a mistake or follows a 'dead end' line of thought?
8. What does my body language communicate?	Do I communicate interest/acceptance/frustration/disapproval ...? How does my body language change through the lesson?

What next?

Having investigated what is actually going on in your classroom, take a look at the relevant sections below and see what would help you develop your classroom culture further.

ASPECT ONE

1. Who does all the talking in whole-class parts of the lesson?

How did this turn out? As teachers we are very good at talking! You will not be alone if you talked for a good percentage of the time in your video. Sometimes we find it hard to let the students have a go and then develop their thinking from there as we are worried they might get stuck, waste their time or lose confidence. We need to give them the confidence that part of the mathematical process is getting stuck and learning from ‘dead ends’ or things that don’t quite work. To explore this aspect further take a look at Number 7 in this article.

Idea to try

• Look for problems that require little explanation to start yet are rich in thinking. For example, you could try How Would We Count? at Stage 1 or Stringy Quads at Stage 2. Try these out and see if that helps you to talk less!

• Give the students 5 minutes to explore the problem and see how they might get started. Then discuss it together as a class.

• Use our games that have a video clip to show the students the game being played: Dotty Six or Strike It Out. Then they can work out the rules through discussion together, rather than you telling them and then making sure that they understand them! This is great for developing their mathematical thinking skills as well as enabling you to talk less. You can develop this idea further by playing any new game under the visualiser with a Teaching Assistant or student so that the students can then try working out the rules. Not got a visualiser? Then use large equipment and gather the students round.

• Discipline yourself to only make a comment on a student’s answer to your question after another student has responded to clarify what was said, ask a question or take the thinking further.

• You may like to ask the students to explain their thinking so far to the rest of the class and then take questions from their peers rather than you, as the teacher, intervening.

Do extend this list with your own ideas or ask other colleagues what they do. This would be a great discussion in a staff meeting.

Digging deeper

Explore the article on Groupworthy tasks. Using these tasks also will also help us talk less.

The article includes a CPD activity that focuses on the value of Groupworthy tasks in developing students' mathematical thinking. It includes a look at how such tasks can help students learn how to persevere and use a 'dead end' to foster a new way of working or a new possible solution to trial.

ASPECT TWO

2. What questions do I ask?

Different questions work well at different stages of the lesson and statements can also be useful, such as 'tell me how you know that'. Below are some examples of generic questions that can be used to guide students through a problem, and at the same time prompt higher levels of thinking.

We can use these questions to guide the students through problems while stimulating their mathematical thinking and gathering information about their knowledge and strategies. As we know, assessment for learning is key to helping us to support students' to move forward in their learning. In Vygotskian terms this enables us to work in the students' Zone of Proximal Development (ZPD) that he saw as the most effective way of promoting learning.

Ideas to try

The questions are grouped in three different ways: stage of the lesson, level of thinking and mathematical skill. See which you find helpful.

Choose one of the three sets of questions and try dividing the questions into three categories:

- questions you use a lot
- questions you use occasionally
- questions you never use.

What do you notice about your categories?

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Add your favourite questions to the list.

Try developing a school set.

You may like to take one or two that are new to you or that you use occasionally and see if you can embed them in your classroom over the next two weeks.
Alternatively you may like to work on a stage of the lesson where you would like to develop your questioning.

This could be a good opportunity to work with a colleague to develop your questioning by sharing ideas, team teaching or by getting involved in some lesson study.

The Questions

A. Stage of the lesson

Here the questions are grouped into four main categories according to the stage of the lesson (Badham, 1994)

Starter questions

These take the form of open-ended questions that focus the students' thinking in a general direction and give them a starting point.

What have we done before that is like this?
How could you sort these ...?
How many ways can you find to ...?
What happens when we ...?
What can be made from ...?
How many different ... can be found?

Questions to stimulate mathematical thinking

These questions assist students to focus on particular strategies and help them to see patterns and relationships. This aids the formation of a strong conceptual network. The questions can serve as a prompt when students become 'stuck'. (Note: we can be tempted to turn these questions into instructions, which is far less likely to stimulate thinking and removes responsibility for the problem solving from the student).

What is the same?

What is different?
Can you group these ... in some way?
Can you see a pattern?
How can this pattern help you find an answer?
What do think comes next? Why?
Is there a way to record what you've found that might help us see more patterns?
What would happen if....?

Assessment questions

Questions such as these ask students to explain what they are doing or how they arrived at a solution. They allow the teacher to see how the students are thinking, what they understand and what level they are operating at. Obviously they are best asked after the students have had time to make progress with the problem, to record some findings and perhaps achieved at least one solution.

What have you discovered?
How did you find that out?
Why do you think that?
What made you decide to do it that way?

Final discussion questions

These questions draw together the efforts of the class and prompt sharing and comparison of strategies and solutions. This is a vital phase in the mathematical thinking process. It provides further opportunity for reflection and realisation of mathematical ideas and relationships. It encourages the students to evaluate their work.

Who has the same answer/pattern/grouping as this?
Who has a different solution?
Are everybody's results the same?
Why/why not?
Have we found all the possibilities?
How do we know?
Have you thought of another way this could be done?
Do you think we have found the best solution?

B. Levels of thinking

We can also look at how these questions can stimulate different levels of thinking, outlined in the table below. You may like to consider the level of question that you are using and how to use more higher-order questions. Are there one or two new questions that you could include in your lesson?

LEVELS OF THINKING	GUIDE QUESTIONS
Memory: recalls or memorises information	What have we been working on that might help with this problem?
Translation: changes information into another form	How could you write/draw what you are doing? Is there a way to record what you've found that might help us see more patterns?
Interpretation: discovers relationships	What's the same? What's different? Can you group these in some way? Can you see a pattern?
Application: solves a problem - use of appropriate generalisations and skills	How can this pattern help you find an answer? What do think comes next? Why?
Analysis: solves a problem - conscious knowledge of the thinking	What have you discovered? How did you find that out? Why do you think that? What made you decide to do it that way?
Synthesis: solves a problem that requires original, creative thinking	Who has a different solution? Are everybody's results the same? Why/why not? What would happen if ...?
Evaluation: makes a value judgement	Have we found all the possibilities? How do we know? Have you thought of another way this could be done? Do you think we have found the best solution?

C. Mathematical skills

Another way of grouping the questions is according to the mathematical skills they encourage.

Exemplifying, Specialising

Describe/demonstrate/show/choose/draw one of …

Is there another?

What's it like?

Give me one/more examples of …

Is … an example of …?
What makes … an example?
Can you find one that doesn't …?
Are there any special ones?

Completing, Deleting, Correcting

What must be added/removed/altered in order to allow/ensure/contradict …?
What can be added/removed/altered without affecting …?
Tell me what's wrong with …
What needs to be changed so that…?

Comparing, Sorting, Organising

What's the same about …?
What's different about …?
Sort or organise these by …
Is it or is it not …?

Changing, Varying, Reversing, Altering

What happens if we change …?
What if …?
If this is the answer to a similar question, what was the question?
Do … in two or more ways.
Which is the quickest/easiest …?

Generalising, Conjecturing

Of what is this an example?
What happens in general?
Can you say why this is special?
What happened here? And here? Can you see a pattern?
Is it always, sometimes, never …?
Describe all possible as succinctly as you can.
What can change and what has to stay the same so that … is still true?

Explaining, Justifying, Verifying, Convincing, Refuting

Explain why …
Give a reason (using or not using…)
How can we be sure that …?
Tell me what is wrong with …
Is it ever false that …? (always true that …?)

How is … used in …?
Explain the role/use of …

Adapted from Jeffcoat, M., Jones, M., Mansergh, J., Mason, J., Sewell, H. and Watson, A. (2004) Primary Questions and Prompts. Derby: Association of Teachers of Mathematics. (See also Watson, A. & Mason, J. (1998) Questions and Prompts for Mathematical Thinking. Derby: Association of Teachers of Mathematics.)

ASPECT THREE

3. Who answers the questions?

I wonder what you discovered when you took a look at your classroom? Was it a range of students who answered the questions or did certain students repeatedly answer?

Ideas to try

• Give 5 seconds wait time before allowing the students to respond to a question. Have you tried using your fingers to count to five behind your back? It is surprising how long that takes! You may well find that this gives more students time to think of their answer and respond.

• Encourage the students to become fluent with the mathematical vocabulary. Students learn to join in conversations by hearing what others are saying, listening to how words are being used and 'playing around' with those words themselves. This means that some modelling of talk is useful – between you and your Teaching Assistant, you and a puppet or you and one of the more articulate students in the class.

• Capture key words and phrases that you hear students using as they talk and put them up on your mathematics 'talk wall' or other display to support the students to use those words. Putting the words inside ready-cut out laminated, speech bubbles can be very effective and create an appealing and interactive display.

• Play dumb! You can stimulate some talk by joining in with a pair/group of students and 'playing dumb'. For example, make a deliberate mistake and see how the students respond.

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Digging deeper

Take a look at how these are ideas can be used in the Dotty Six game.

ASPECT FOUR

4. How well do I listen to the students' answers and seek to understand what they are saying?

Listening carefully to what the children actually say is sometimes harder than we realise. We may not hear clearly what they say as we may be expecting them to give us a fixed answer that we have pre-determined – this can be called, 'guess what is in the teacher's head!' We need to be ready to be open to their answers and be curious to understand what they are trying to say.

Ideas to try

• Be curious about what the student was saying and ask a clarifying question such as, 'so what you are saying is ...?' You could alternatively invite the students to tell a partner what they think their peer said. This is also useful if their answer is rather jumbled or rambling. Our temptation, in this case, is to rephrase it, reorganise it and repeat it back to the class in what we consider to be its new, improved form. We may hear ourselves saying something like,

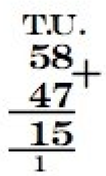
'Thank you, Elspeth. What Elspeth said was ...'

See what happens, if instead, you check with the student, 'Elspeth' if you have heard what they said correctly by saying something such as, 'I think what you said was ... Am I right?' When saying what you thought they said try and use the same words that they used.

• Resist the urge to finish their sentence for them with what you think they want to say or what you hope they will say! See what happens if you just repeat back to the student what they have said, using the same words they have used, and see if that helps them to finish the sentence. Doing this for a number of weeks can help them gain confidence to finish what they wanted to say rather than what we thought they might want to say!

• Avoid making assumptions about what the student is saying. Check it out! It will help you to support the student’s learning much more effectively.

Here’s an excerpt from Bernard’s article What’s All the Talking About? that illustrates how easy it is to make an assumption:

> I was visiting a teacher in her classroom and was invited to observe the students working. I arrived at a 9-year old boy's table who had just written this,
>
> 
>
>
> When subsequently sharing this experience with teachers they were quick to identify the mistakes that the boy had made and which indicated his lack of understanding of place value. But at the time I spoke to the boy, the following took place,
>
> Me: "Tell me about this."
>
> Boy: "Its easy 7 and 8 make 15, put down the 5 and carry 1. Five 4 five and 1 make 15.
>
> Me: "What does this say?" pointing to the 58 and wanting to discover how he is thinking about these numbers.
>
> Boy: "It's a 5 and an 8."
>
> Me: "Tell me about these" pointing to the T and U.
>
> Boy: "That's Tens and Units," said in an annoyed voice, he seemed to want to copy down the next similar sum from the work card.
>
> I wrote 58 on a piece of scrap paper, as I wanted to see what he would call it. He said that it was another 5 and an 8. I tried to think of another situation in which he may have seen tens and units. I thought of bus numbers.
>
> Me: "Suppose a bus went by and on. . . . " he interrupted me.
>
> Boy: "It'll be a fifty-eight."

Me: "Could we call this fifty-eight?" pointing back to the sum.

Boy: "Yes, if you like" said nonchalantly.

Me: "And this?" pointing to the 47.

Boy: "It'd be forty-seven. Do you want me to add it?"

Me: "Well yes."

Boy: "40 and 50 is 90. 7 and 8 is 15. The answer is 105." This was said as quickly as most people could say it.

So you can imagine the response of the teachers when they heard this, previously thinking he lacked much understanding of tens and units, and then realising he could do the calculation so quickly in his head! From that moment on I vowed to not make assumptions about what students can or cannot do.

ASPECT FIVE

5. What do I do with the students' answers?

I wonder what you discovered from your video and whether you were surprised by what you did. As teachers we often feel the need to speak after every student has spoken and offer some comment. After all it is our job!

Ideas to try

• Keep quiet! Let a student speak next with a question, further idea or comment.

• Accept every answer as a useful contribution to the discussion by simply saying, 'thank you' rather than repeating the contribution or giving a value judgement such as 'good answer' or 'fabulous idea'.

• Follow it with another question – see our list of suggestions. You might like to see if you can add to this as a staff and make a whole-school list of great follow-up questions.

Some follow-up questions to start your list:

- Are you sure?
- Convince me.
- Show me how you know that.
- I am curious to know why you chose that one.
- I would choose this one ... Are we both right?
- What could happen if ...?

ASPECT SIX

6. How do I facilitate the learning?

This will depend on how you structure the lesson and how you encourage the students to engage with the problem.

Idea to try

Some helpful ideas for this are exemplified using the Make Those Bracelets task below.

You might like to try out this activity with a colleague and then plan how you would teach it to a suitable class or group of students. See if you can arrange to have a go at teaching it with that class, or a small group of four from that class, and reflect on it with your colleague afterwards. It would be great if you could both teach the lesson together, with different classes or with a small group of four each. Videoing the lesson could be very helpful in digging deeper into the learning.

Take a look at the Teachers' Notes and also at the notes below when preparing to use the problem with a class or small group. If it would help you to have the solutions to the problem before you embark on it with the students then do email us for them.

Decide what you want to focus on, in terms of your teacher behaviour, such as talking less, what I do to encourage the students' own thinking, what questions I ask or how I respond to their questions.

Be clear in your lesson plan what you are going to try out in the light of your focus above – particular questions, clarifying what the students say etc.

Reflect on the lesson with your colleague and decide what you want to try differently in your classroom as a result of trying out the bracelet investigation.

Making Those Bracelets: possible teacher behaviours

Introducing the problem
The students work in groups of four on this problem.

The African context of this problem is important. Emphasise that boys, girls and parents are using a variety of beads to make colourful bracelets.

Say to the students that they are going to have a go at this by making bracelets with just two different colours of beads. They can use between two and five beads of each of the two colours. The reason for this is that it will help you to focus on the mathematical possibilities, whereas with lots of colours and lots of beads the outcomes are far too numerous to see patterns and make predictions. This is the aim of the activity.

So the particular challenge asks you to find ALL the ways of arranging four, five, six, seven, eight, nine, ten beads on the bracelets each time using just TWO colours. Watch out! You may find you have made some that are the same.

How can you convince others that you have found them all? Try and do this once you think you have got all the possible ways for one of the numbers of beads.
This is a key part of the problem.

Finally, without working with the beads themselves and looking at what happened when you used four to ten beads see if you can predict how many different bracelets you can make if you use six beads of each of the two colours.

Things to avoid saying that will give the game away!

• Make sure you give no clues to the students about how many beads to chose to use – they need to decide themselves how to work with the challenge of four, five, six, seven, eight, nine and ten.

• Let the students discover the potential repeats in flipping the bracelets over.

Main problem-solving time

As each of the groups get started on working on the problem, look and see how they are working. Are they working as a team and dividing up the ways of exploring the bead arrangements in a systematic way or are they working individually?

As the group works on the problem, the following prompts might be helpful:

• What have you done so far? (This is a useful starting point so you can begin to understand how the group has approached the problem).

• Remind them that the beads need to be of two colours only and between four and ten altogether.

• Encourage them to explore whether the arrangements they are making are all unique.

• Ask them how they are going to work out how they have found all of them.

• How will you remember what you have done so far? (This is a good way to encourage some form of recording. Groups are likely to have different ways of recording and some may be new to us).

SSSSSHHH ... don't say this ... top tip! ... If the students record their answers in a line rather than in a connected bracelet form it is very easy for them to get repeats without realising it. For example, RRBRBB is the same as RBRRBB (starting in the middle and going backwards – i.e. the bracelet has been flipped over and any place can be a start as it is a circle).

Encourage them to see if they can see a pattern for the number of arrangements they have found.

Watch out for those groups that are working systematically.

Watch out also for those students who are using logical reasoning to decide if arrangements are unique – this one is the same as that because if I ...

Taking it further

When they feel confident about convincing others of their results for four-ten beads, remind them of the final part of the problem. This challenge is to predict the number of different bracelets that could be made with six beads of each colour without 'making' each one.

It is unlikely that many will get this far in a single lesson. You may like to have the group for an afternoon or devote two lessons to it.

ASPECT SEVEN

7. How confident are the students to take a risk, try out ideas, make mistakes?

Students need to feel safe to explore their ideas in the knowledge that it will be fine if they get it wrong: in fact, getting it wrong will be positively welcomed as this could well show us something about the problem. How do we achieve this?

Idea to try

To explore this further, by yourself or as a school, use the article I'm Stuck!. Here Liz offers two possible problems for you to try to see how it feels and also shares her own experiences. Choose one of the problems. After you have tried the problem think about how your own experiences could influence what you do in the classroom. The questions below may help you reflect on this.

Your experience
What was important to you?
How did you start?
How did you feel about the colleague who seemed to whizz ahead and know what to do?
What helped you refine your ideas?
What did you do when you got stuck?
What inspired you to ask further questions?

In the classroom
What might you do differently?
What might you do more of/less of?
How might you support the less confident learners?
How could you support the development of thinking?

Digging deeper

Explore the article on Groupworthy tasks. The article includes a CPD activity that focuses on the value of Groupworthy tasks in developing students' mathematical thinking. It includes a look at how such tasks can help students learn how to persevere and use a 'dead end' to foster a new way of working or a new possible solution to trial.

(This article is also referred to in Aspect One.)

ASPECT EIGHT

8. What does my body language communicate?

Our body language can speak volumes without us necessarily being aware of its message! Were you suprised as you watched the video of your lesson?

Idea to try

Identify one challenge for yourself, based on your observations from the video of your lesson. For example, you may decide that you will try to cross your arms less frequently or that you will always try to make eye contact with whoever is speaking. Share your 'target' with your Teaching Assistant and ask s/he to monitor that particular aspect of body language over a certain period during a future lesson. Make time to listen to his/her assessment and do that same thing later in the term.

Further Reading

1. Engaging Students, Developing Confidence, Promoting Independence
2. Developing a Framework for Mathematical Enrichment
3. Going for Games
4. Cultivating Creativity

References

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