

# THE DEVELOPMENT OF FOUNDATIONAL NUMBER SENSE IN ENGLAND AND HUNGARY: A CASE STUDY COMPARISON

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*Foundational number sense - being able to operate flexibly with number and quantity - is a predictor of later mathematical achievement. In this paper, drawing on lessons on number sequences to grade 1 children, we examine how two teachers, one English and one Hungarian, construed locally as effective, created opportunities for children to develop foundational number sense. The Hungarian teacher, in ways typical of that country's mathematics teaching tradition, offered frequent and coherent opportunities for students to develop foundational number sense. The English teacher, working in a tradition whereby interactive technology increasingly mediates classroom discourse, offered few and less coherent opportunities, masked by the teacher's frequent attention to display features of the technology.*

## INTRODUCTION

Described as a “traditional emphasis in early childhood classrooms” (Casey et al 2004: 169), the quality of young children’s number sense is a key predictor of later mathematical success, both in the short (Aunio and Niemivirta, 2010) and the longer term (Aunola et al, 2004). Consequently, particularly as number sense deficits tend to lead to later difficulties (Jordan et al., 2007), the development of children’s number sense “is considered internationally to be an important ingredient in mathematics teaching and learning” (Yang and Li 2008: 443). Importantly, without appropriate intervention children who start school with limited number sense are likely to remain low achievers throughout their schooling (Aubrey et al., 2006).

### Defining foundational number sense

Our understanding of number sense is that it comprises two related but distinct concepts that the literature typically conflates. The first, which we label foundational number sense (FNS), concerns the number-related understandings children develop during the early years of formal instruction. The second, which we have labelled applied number sense, draws on the first and concerns the number-related understanding necessary for people to be mathematically competent and functionally effective in society. In this paper, while remaining mindful of the latter, we focus on the former.

For the purpose of this paper we define FNS as the ability to operate flexibly with number and quantity. It develops over time “as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not

limited by traditional algorithms” (Sood and Jitendra, 2007, p. 146). It comprises, so our understanding of the literature informs us, several elements, among which are:

Awareness of the relationship between number and quantity (Berch, 2005; Clarke and Shinn, 2004; Van de Rijdt et al., 1999; Griffin, 2004).

Understanding of number symbols, vocabulary and meaning (Clarke and Shinn, 2004; Van de Rijdt et al., 1999; Malofeeva et al. 2004; Yang and Li, 2008).

Systematic counting, including notions of ordinality and cardinality (Gersten et al., 2005; Griffin, 2004; Malofeeva et al. 2004).

Awareness of magnitude and comparisons between different magnitudes (Gersten et al., 2005; Griffin, 2004; Ivrendi, 2011; Jordan et al., 2007; Malofeeva et al. 2004).

An understanding of different representations of number (Ivrendi, 2011; Jordan et al., 2007; Yang and Li, 2008),

Competence with simple arithmetical operations (Berch, 2005; Ivrendi, 2011; Yang and Li, 2008).

An awareness of number patterns including recognising missing numbers (Berch, 2005; Clarke and Shinn, 2004; Jordan et al., 2007).

In this paper, as an introduction to comparative research in the field, we examine the question, what opportunities do two teachers, in different cultural contexts and considered locally as effective, create for their students to acquire FNS?

## DATA COLLECTION AND ANALYSIS

In addressing our question, we compare excerpts from two lessons taught to grade one children in England and Hungary. The lessons from which they were drawn were each part of a wider collection of videotaped lessons gathered independently of each other. The English lesson derived from the second author’s PhD case study examination of primary mathematics teachers’ enactment of whole class teaching. The Hungarian lesson derived from a study of exemplary Hungarian primary mathematics teaching undertaken by the third author to inform curriculum development activities in England. Both teachers were construed against local criteria as effective in the manner of the Mathematics Education Traditions of Europe project (Andrews, 2007). Thus, the two data sets were not only serendipitously available but also appropriate for comparative analysis. Both teachers, with microphones, were video-recorded in ways that would optimise the capture of their actions and utterances. Both data sets entailed repeated observations of a small number of case study teachers over a period of several months in order to ensure a sense of the typical lesson. The Hungarian lessons were supported by a home-based English-speaking colleague providing a contemporary translation augmented by the first author’s sufficient understanding of Hungarian to be able to follow much of the discourse of a mathematics classroom. In other words, while the two sets were collected independently, they were gathered in similar ways and amenable to similar

analyses. The excerpts analysed here were based on the teaching of number sequences. This topic was chosen because, among the various FNS-related components, it was the only one addressed explicitly in both sets of lessons. Moreover, our belief was that number sequences would provide more opportunities for the incidental teaching (Radwan, 2005) of the other FNS components. With respect to analysis each excerpt was viewed simultaneously and repeatedly by all three researchers. This led to our seeing not only that each excerpt comprised three distinct phases, which frame our analyses, but also which components of foundational number sense were addressed, both implicitly and explicitly.

## RESULTS: THE ENGLISH EXCERPT

This excerpt derives from a lesson taught to 6 and 7 year olds on number sequences and patterns. It began with the teacher, Sarah, reminding her children about previous work on sequences, how odd and even numbers create an alternating pattern in the number system, and informing them that today they would be looking at some more sequences and number patterns. The 5 x 10 grid below was displayed on the interactive white board – a resource typical of English classrooms - with the cells containing the first six even numbers shaded. Sarah asked if anyone could explain the pattern. One child responded by saying that they all end in 0, 2, 4, 6 or 8, after which Sarah commented that they are all even. Next she asked about the column patterns and, after a suggestion that the numbers in each column end in the same digit, Sarah accepted the suggestion that the column pattern goes odd even odd. At this point, exploiting the whiteboard’s software, she displayed the odd columns in one colour and the even in another.

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 33 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 43 | 34 | 45 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 53 | 44 | 55 | 46 | 47 | 48 | 49 | 50 |

Commentary: In these first few minutes several FNS categories appeared to have been addressed. For example, Sarah’s explicit emphases seemed very much geared towards inducting her children into an awareness of number patterns alongside a clear expectation that children recognise number symbols and vocabulary.

Next, Sarah put up another 5 x 10 grid but with no coloured cells. Beneath the grid was the following:

1, 4, 7, 10, 13, \_\_, \_\_

She asked how other numbers in the sequence could be found and invited her class to look at the number grid. Having evoked no responses Sarah tapped, in turn, each of the five numbers to change the colour of their cells and create the image below. At

this point she commented that the grid looked different from that of the previous problem and asked *have I done it wrong?* Different students offered, tentatively, both negative and positive responses, which went without comment.

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 33 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 43 | 34 | 45 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 53 | 44 | 55 | 46 | 47 | 48 | 49 | 50 |

Next, Luke raised his hand and the following ensued:

- Sarah        Luke?
- Luke         Sixteen
- Sarah        Why sixteen?
- Luke         Because you're adding on three
- Sarah        Because it's adding on three isn't it (she taps the cell to change the colour)...  
What's going to be the next one? Isla?
- Isla         Nineteen
- Sarah        What's going to be the next one? Ian?
- Ian         Twenty-two
- Sarah        Twenty-two (she taps the cell). And the next one? (more hands go up this time) Rachel?
- Rachel      Twenty-five

Commentary: In the above is evidence of different aspects of FNS. The four students were clearly extending the sequence given them. Also, the assertion that they were adding on three indicated an engagement with simple arithmetical operations and, of course, they were still being made aware of the relationship between symbols, their vocabulary and meaning. By now the grid looked as below.

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 33 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 43 | 34 | 45 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 53 | 44 | 55 | 46 | 47 | 48 | 49 | 50 |

- Sarah        Twenty-five (she clicks on the cell, which becomes red) Can anybody see any colour patterns coming out of this?
- Boy         It looks like a bit like stairs!

Sarah Rosie? (Rosie says something inaudible so Sarah walks to the middle of the room to hear what Rosie says)

Sarah Yes, they're going diagonally aren't they. Yes when we extended this pattern we started to see that. There's a diagonal pattern made by the squares coloured in. Now, if Rosie's right then this one's (she taps 34, which turns red) going to be in our sequence. I'm going to fill it in and then count on three each time. One, two, three (she taps 28, which change it to red). One, two, three (she taps on 31). One, two... (she points to 34, which is already red, and faces the class)

Children Threeee (a few children shout out)

Commentary: In this closing episode Sarah appeared to be encouraging systematic counting from an ordinal perspective as well as further opportunities for children to recognise number symbols, vocabulary and meaning. Thus, over the whole excerpt, Sarah seemed to have addressed four of the seven FNS categories.

## RESULTS: THE HUNGARIAN EXCERPT

This excerpt was taken from a sequence of lessons focused on children's coming to know and work with integers to 20. The lesson began with Klara, having written on the board prior to the start of the lesson, the configuration shown below.

|   |   |   |    |   |   |   |   |   |
|---|---|---|----|---|---|---|---|---|
| — | — | — | —  | — | — | — | — | — |
| 3 | 7 | 6 | 10 | — | — | — | — | — |

Having been invited to do so, the class read out the numbers in unison as Klara pointed to each in turn. Next, moving from left to right, she invited volunteers to explain how each number could be derived from the one preceding it. Students volunteered that the first operation was add four, followed by subtract one and add four. With each offering Klara wrote the operation underneath, as shown below, before inviting predictions as to what operation would be expected next. Eventually, after several contributions, the table was completed.

|    |    |    |    |   |   |   |   |   |
|----|----|----|----|---|---|---|---|---|
| —  | —  | —  | —  | — | — | — | — | — |
| 3  | 7  | 6  | 10 | — | — | — | — | — |
| +4 | -1 | +4 |    |   |   |   |   |   |

Commentary: During this period it seems to us that Klara had encouraged several aspects of FNS. Firstly, an introduction during which children were invited to recognise and read the numbers on the board was focused on the recognition of number symbols and their vocabulary. Secondly, in the ways in which successive numbers were identified, Klara was addressing simple arithmetical operations. Thirdly, the episode was explicitly focused on an understanding of number patterns.

Next, Klara produced some cards, each of which had a letter written on it. She announced that she was going to ask questions, the answer to each would be one of the numbers in the sequence. Each correct answer would yield a letter to spell a word that would tell the class where it would be going in the story of this particular day. The following reflects the first minute of next five minutes of discourse.

- Klara            So my first statement is... please look only at the numbers on the board... I am thinking of the largest one-digit number. Balasz?
- Balasz           Six (Pupils protest)
- Klara            Look at the sequence again, and please correct yourself.
- Balasz           Seven
- Klara            Look at the number line... Ferenc?
- Ferenc           Nine
- Klara            That's right. So I will give you a reward for the nine (Klara placed a card with the letter Í on the board above the number nine). The next number I am thinking of... You mustn't look behind you (Referring to a picture on the back wall) is the value of the black stick in our collection. Perszi?
- Perszi           Eight (Pupils protest)
- Klara            (to Perszi) Look around, the others don't agree with you... Mara?
- Mara            Seven
- Klara            Let's see who's correct. (They all look at the back wall, where they can see the members of the Cuisenaire rod collection and their values)
- All                (In chorus) Mara was correct (at which point Klara placed a card with the letter Á above the seven).

The lesson continued with Klara asking a different form of question for each number in the sequence. These included statements like, a number two smaller than nine, the largest two digit number, the smallest one digit number, a number whose digits add up to 4, and so on. In each case at least one child was involved in publicly responding to the questions posed. Eventually, as shown in figure 4, the following emerged with only the number ten left without its corresponding letter.

B            Á            B            \_            Í            N            H            Á            Z

|    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|
| 3  | 7  | 6  | 10 | 9  | 13 | 12 | 16 | 15 |
| +4 | -1 | +4 | -1 | +4 | -1 | +4 | -1 |    |

Commentary: In this phase several FNS categories were addressed. The largest one-digit number discussion encouraged not only recognition of number symbols, their vocabulary and meaning but also awareness and comparison of magnitude. In considering a Cuisenaire rod, Klara addressed not only an understanding of different representations of number but also an awareness of the relationship between numbers and quantities. In fact, every statement seemed to address an element of FNS. For example, a number two smaller than nine is a different representation of seven. Similarly, several statements involved simple arithmetical operations.

Having identified all the letters bar the one linked to ten Klara passed responsibility to her students and invited them to offer statements appropriate to that number. This led to the following:

- Mara: It is the bigger neighbour of the number 9.
- Ildikó: It is the smallest 2-digit number.
- Csaba: It is the smaller neighbour of the number 11.
- Gabor: The sum of its digits is 1.
- Judit: Even number.
- Klara: Have we got anything else?
- Zsolt: It is the sum of the 1 and 9.
- Klara: Yes, the sum of the 1 and 9... and who knows the letter in my hand?
- All: Sz (the juxtaposition of s and z in this manner is, in Hungarian, an alphabetic letter with a sound similar to the s in sun)
- Klara: Yes, and where are we going today?
- All: Bábszínház. (Puppet theatre)

Commentary: In this final episode was further evidence of Klara's promotion of FNS. For example, both understanding of different representations of number and recognition of number symbols, their vocabulary and meaning was, we believe, implicit in all contributions. Simple arithmetic could be seen in Gabor's statements that the sum of ten's digits is one and Zsolt's suggestion that ten is the sum of one and nine. Awareness of number patterns was implicit in Judit's even number suggestion, while awareness of magnitude was implicit in Mara's bigger neighbour of nine and Eva's smaller neighbour of eleven.

## DISCUSSION

In this paper we have attempted to show how opportunities for students to acquire FNS played out in two culturally different classrooms. Apart from explicit focus of

the excerpts – number sequences - neither teacher appeared to focus explicitly on the development of FNS. However, through an analysis of such snapshots we can examine the extent to which the development of foundational number sense is an integral, albeit implicit, component of children’s early school experiences of mathematics. Moreover, since both teachers were construed locally as effective, such snapshots may offer insight into how teachers, in different cultural contexts, have been conditioned - by their experiences as learners of mathematics, their professional training and subsequent career opportunities - to address such issues.

The analyses above, summarised in table 1, indicate both similarities and differences in the ways in which foundational number sense was addressed. In respect of similarities both teachers addressed several categories, with Klara addressing six of the seven categories and Sarah four. Both encouraged, throughout their respective excerpts, students’ recognition of number symbols, vocabulary and meaning. Both encouraged the awareness of number patterns and missing numbers and both exploited simple arithmetical operations. In respect of differences Klara addressed three categories, the relationship between numbers and quantities, comparisons of magnitude and different representations of number that Sarah did not, while Sarah was seen to address systematic counting when Klara did not.

|  | Sarah’s episodes |   |   | Klara’s episodes |   |   |
|--|------------------|---|---|------------------|---|---|
| Relationships between numbers and quantities |                  |   |   |                  | X |   |
| Number symbols, vocabulary and meaning       | X                | X | X | X                | X | X |
| Systematic counting                          |                  |   | X |                  |   |   |
| Comparisons of magnitude                     |                  |   |   |                  | X | X |
| Different representations of number          |                  |   |   |                  | X | X |
| Simple arithmetical operations               |                  | X |   | X                | X |   |
| Number patterns and missing numbers          | X                | X |   | X                | X |   |

Table 1: the distribution of the categories across the excerpts’ episodes

However, while it is clear that both teachers encouraged various aspects of FNS as part of the incidental learning of their lessons it seems to us that Klara’s was a more didactically complex encouragement than Sarah’s. Put crudely, Klara addressed, on average, four categories of foundational number sense per episode while Sarah addressed barely two. Klara encouraged mathematical reasoning, while Sarah seemed to subordinate such reasoning to an examination of the coloured patterns on the interactive whiteboard. This latter expectation, it seems to us, appeared not only a distraction from children’s learning of mathematics but indicative of a more general problem for teachers working in technology enhanced classrooms (Muir-Herzig, 2004; Wang and Reeves, 2003). Moreover, if number sense develops gradually as a

result of exploring and visualizing numbers in different contexts (Sood and Jitendra, 2007) then Klara's practice seems more likely to succeed than Sarah's.

Interestingly, both teachers' practices find some resonance with earlier studies of mathematics teaching in the two countries, albeit at the level of the upper primary classroom rather than the lower primary. Andrews (2009) found Hungarian teachers exhibiting didactical sophistication in their encouragement of cognitively demanding but coherent learning outcomes. The same study found English teachers exhibiting relatively unsophisticated didactical practices in their promotion of substantially more modest and less coherent goals. That is both Klara's and Sarah's practices appeared commensurate with that of their compatriot colleagues working in later phases of schooling. Thus, the limited evidence of this study indicates that teachers working in the first years of schooling, teachers defined locally as effective, behave in ways similar to their compatriots and that the Hungarian tradition seems more likely to facilitate a secure FNS than the English. That being said, clearly more research in this particular field is necessary if we are to understand more fully how teachers induct their learners into this essential mathematical prerequisite.

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