Developing Excellence in Problem Solving with Young Learners

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Becoming confident and competent as a problem solver is a complex process that requires a range of skills and experience. As teachers we can support this process in three principal ways:

- Through our choice of task
- Through structuring the stages of the problem-solving process
- Through explicitly and repeatedly providing children with opportunities to develop key problem-solving skills.

Choice of task

NRICH offers a wide range of rich tasks to engage learners in the problem-solving process. We want all our tasks to be used in such a way that they enable learners to explore and work from their own level of understanding, and then build on this towards new understandings. We believe that this approach offers the opportunity for rich, embedded learning.

Some of our NRICH tasks are simple games that can be played time and time again to develop numerical fluency as well as problem solving and reasoning, such as Spiralling Decimals https://nrich.maths.org/10326, and Totality https://nrich.maths.org/1216 (see also Developing Number Fluency – What, Why and How https://nrich.maths.org/10624), whilst others may be non-routine problems that have a specific solution (such as Shape Times Shape https://nrich.maths.org/5714). Some may take a short time, like Shut the Box https://nrich.maths.org/6074, whilst others may intrigue and challenge over more than one lesson, like Dice in a Corner https://nrich.maths.org/8586.

The key is to be clear how you want to use a particular problem to support the development of the children's skills to problem solve. It is tempting to choose a problem that relates to the mathematical content that you have been working on in class. However, whilst children need to be fluent with the mathematical content demands of any problem they tackle, it may be more productive to choose a problem that builds on a specific element of problem solving that you are working on as a class, and uses content that they are very familiar, and more confident, with. The Primary National Strategy (May 2004) suggested that there are five different types of problem:

- Finding All Possibilities (Half Time https://nrich.maths.org/7408)
- Logic (Teddy Town https://nrich.maths.org/108)

- Visual (Baravelle https://nrich.maths.org/6522)
- Rules and Patterns (Ip Dip https://nrich.maths.org/7185)
- Word Problems

This is one way of trying to divide up problems into categories. However, it is clear that not all problems fit neatly into just one category and we may debate the categories. What this idea of different types does offer, however, is a way of giving children the experience of a similar type of problem over a number of weeks so that they can gain some proficiency. For example, you might like to explore a number of problems that encourage the children to Find All Possibilities – see our collections:
Lower Primary https://nrich.maths.org/9779
Upper Primary https://nrich.maths.org/9784

The stages of the problem-solving process
The problem-solving process can usually be thought of as having four stages:

- Stage 1: Getting started
- Stage 2: Working on the problem
- Stage 3: Digging deeper
- Stage 4: Reflecting

Although the stages are numbered, problem solving is not necessarily a linear process. We might, for example, reflect on what we have done so far and return to working more on the problem before digging deeper.

We can helpfully spend time with children concentrating on one of these stages explicitly, in turn, as they learn to become confident problem solvers.

Stage 1: Getting started will mean offering them strategies to help them engage with the problem. These could be prompts such as:

- Tell me/a partner what you think the problem is about.
- What would help you understand the problem?
- You might like to draw a diagram, act it out or represent it with a model.
- Could you try something out and see what happens?
- What other problems have you seen that are ‘a bit like’ this one?
- What mathematical skills have you got that could be helpful here?
- Try making a simpler case to get an idea of how the problem works.

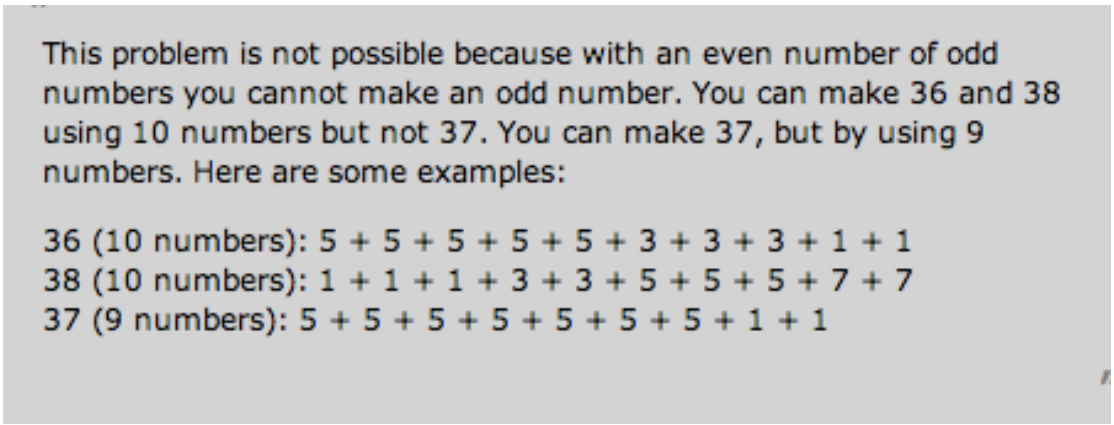
Stage 2: Working on the problem will usually involve using one or several problem-solving skills such as:

- Trial and improvement
- Working systematically (and remember there will be more that one way of doing this: not just the one that is obvious to you!)
- Pattern spotting
- Working backwards
- Reasoning logically
- Visualising
- Conjecturing.

These problem-solving skills are in a random order, although the first two, trial and improvement and working systematically, are key skills that will support children to become competent as problem solvers.

The children will benefit from becoming proficient in each of these skills and working on one of them as a key focus in a lesson or series of lessons could be a useful strategy.

Stage 3: Digging deeper usually happens when the problem has been explored and then it is possible to look for generalisations and proof. Here's an example from our NRICH problem Make 37: https://nrich.maths.org/1885


This problem is not possible because with an even number of odd numbers you cannot make an odd number. You can make 36 and 38 using 10 numbers but not 37. You can make 37, but by using 9 numbers. Here are some examples:

36 (10 numbers): 5 + 5 + 5 + 5 + 5 + 3 + 3 + 3 + 1 + 1
38 (10 numbers): 1 + 1 + 1 + 3 + 3 + 5 + 5 + 5 + 7 + 7
37 (9 numbers): 5 + 5 + 5 + 5 + 5 + 5 + 5 + 1 + 1

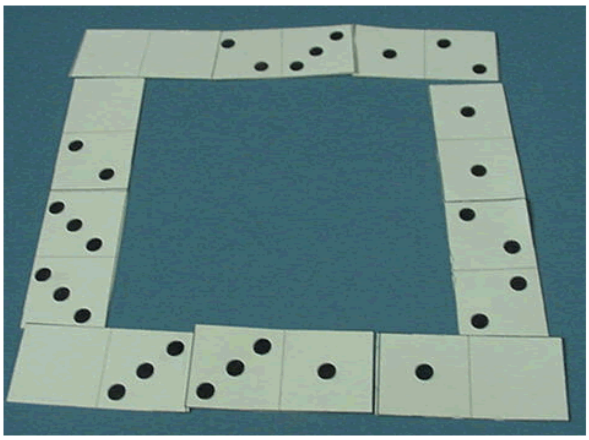
Stage 4: Reflecting is the part of the problem-solving process where we support the children to:

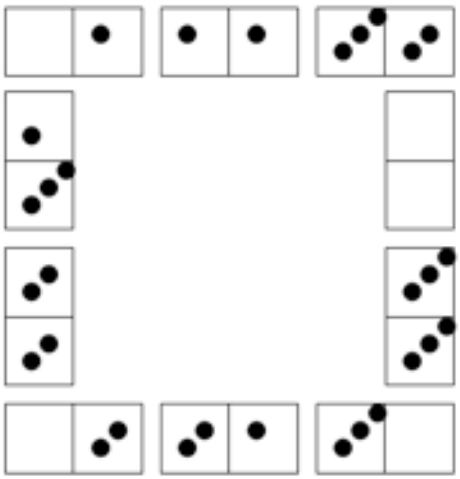
- learn to explain their findings so far in the context of the problem
- explain their solution both verbally and in writing

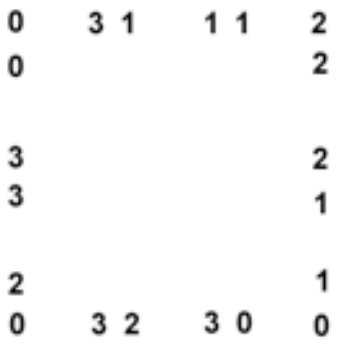
- evaluate their method and compare different strategies.

They may need some support to explain succinctly, use words such as ‘because’ and to use the appropriate mathematical vocabulary correctly. This all takes time, attention and practice.

Written recording could be in the form of a photograph, diagram or written explanation. Here are some children’s solutions to our NRICH problem Domino Squarehttps://nrich.maths.org/146 that illustrate how recording may develop:







Children will need support to develop their proficiency with written recording. Here's an example of a child explaining their thinking for Eggs in Baskets https://nrich.maths.org/2002. She has used a trial and improvement approach.

> If there was 1 egg in the brown basket, no eggs in the red basket and 3 eggs in the pink basket, that would only make 4 eggs.
> If there were 2 eggs in the brown basket, 1 egg in the red basket and 4 eggs in the pink basket, this only makes 7 eggs.
> So finally, if there were 3 eggs in the brown basket, 2 eggs in the red basket and 5 eggs in the pink basket, there would be 10 eggs altogether.

You can read more about types of recording in our article at https://nrich.maths.org/9871.

As children reflect on the problem-solving adventure, they will need to be supported to compare different strategies that were used to solve the problem in order to consider the efficiency of the method and the elegance of the solution. This will enable them to see how they might refine their own methods or adopt a different one next time they encounter a similar problem.

The skills needed for a problem-solving task
By this we mean the problem-solving skills listed above in Stage 2: working on the problem. It will help the children become fluent in these if you take every opportunity to explicitly talk about them and use the appropriate language when they occur in games or larger problem-solving activities. You may like to focus on developing one or two at a time.

Our youngest learners can start thinking about 'working systematically' in contexts such as choosing two toppings out of sprinkles, sugar stars or flakes to go on top of iced biscuits they are making. The key question is – 'how do you know that you have got them all?'. This comes after, 'I can find some solutions' and 'I can recognise ones that are the same'. For example, is having sprinkles and sugar stars the same as having sugar stars and sprinkles on top of my iced biscuit?

Children are often quite good at having a random guess as to how to solve a problem. To become fluent at trial and improvement they need to be able to think about how to adapt their first guess so that it is more likely to become a solution rather than throwing the first one out and starting again. When starting to explore Dice in a Corner https://nrich.maths.org/8586children may well put the dice together at random and be surprised when they get the magic total of 18. Those children who are becoming fluent at trial and improvement will then want to adjust the dice to see if they can make 18 in another way, rather than trying another random arrangement.

There are lots of NRICH problems that will help you develop these skills with children. You will find some of our collections at https://nrich.maths.org/8935

Being a competent and confident problem solver is central to the mathematical development of all our learners. It is also the major aim of the mathematics national curriculum in England. This article has detailed the individual elements that teachers can focus on to support children to gain this level of proficiency. We trust you will find it useful and we are always interested in your feedback and experiences as you explore problem solving together with the children in your class.

References

Primary National Strategy (2004) Problem solving A CPD pack to support the learning and teaching of mathematical problem solving. DfES Publications