

About Average

1. 3 numbers, mean = 3

Thus the sum of the three numbers
 $3 \times 3 = 9$

Mode = 2

$$\begin{array}{c} 2, 2, \square \\ \square = 9 - (2+2) = 5 \end{array}$$

Ans: 2, 2, 5

2. 3 numbers, mean = 7

Sum of the three numbers = $7 \times 3 = 21$

Mode = 10

$$\begin{array}{c} 10, 10, \square \\ \square = 21 - (10+10) = 1 \end{array}$$

Ans: 1, 10, 10

3. 3 numbers, mean = 8

Sum of the three numbers = $8 \times 3 = 24$

Median = 10

$$\begin{array}{c} \square a, 10, \square b \end{array}$$

$$a + b = 24 - 10 = 14 \quad \text{--- ①}$$

$$\text{Range } 8: b - a = 8 \quad \text{--- ②}$$

$$\text{①} + \text{②}: 2b = 14 + 8$$

$$b = 11$$

$$a = 14 - 11 = 3$$

Ans: 3, 10, 11

4. 4 numbers, mean = 7.5

Sum of the four numbers = $7.5 \times 4 = 30$

mode = 6 IF 6 appears two times:

Case A : \boxed{a} , 6, 6, \boxed{b}

OR

Case B : 6, 6, \boxed{a} , \boxed{b}

As sum = 30, $a + b = 30 - (6 + 6) = 18$

Case A : Median = 6, not correct

Case B : Median = 7

$$6 + a = 7 \times 2 = 14$$

$$a + b = 18$$

$$a = 8, b = 10$$

mode = 6 IF 6 appears three times:

\boxed{a} , 6, 6, 6

OR

6, 6, 6, \boxed{b}

This case, median = 6 so this is not correct.

Ans : 6, 6, 8, 10

5. 4 numbers, mean = 6

Sum of the four numbers = $6 \times 4 = 24$

$$\text{median} = 6.5 \quad \boxed{a} + \boxed{b} + \boxed{c} + \boxed{d} = 24$$

$$\frac{b+c}{2} = 6.5$$

$$b+c = 13$$

$$a+d = 24 - 13 = 11 \quad \text{--- ①}$$

As range = 11, $d - a = 11$ --- ② ① + ②: $2d = 22$ $d = 11$ $a = 0$
 $0, \square, \square, 11$ ~~No~~

Since all numbers must be possible positive integers, there's no possible answer.

6. 5 numbers, mean = 4

Sum of the five numbers = $4 \times 5 = 20$

IF 3 appears three times:

$$3, 3, \boxed{a}, \boxed{b}, \boxed{c}$$

← Not correct because $a+b = 2$ and so it is not less than 3.

$$\text{① --- } \boxed{a}, 3, 3, \boxed{b}, \boxed{c}$$

$$\text{② --- } \boxed{a}, \boxed{b}, 3, 3, \boxed{c}$$

$$\square, \square, \square, 3, 3 \quad \leftarrow \text{Not correct because range} = 9$$

$$\text{① } a+b+c = 14$$

$$c - a = 9$$

$$b + 2c = 23$$

$$a = c - 9$$

b	c	b+2c	a
3	10	23	1
5	9	23	0
7	8	23	-1

← can't be negative.

$$\text{② } \begin{array}{c|c|c|c} b & c & b+2c & a \\ \hline 1 & 11 & 23 & 2 \\ \hline 3 & 10 & 23 & 1 \end{array}$$

← Not correct because a is bigger than b.

Ans: 1, 3, 3, 3, 10

7. 5 numbers, mean = 4

$$\text{Sum of the five numbers} = 4 \times 5 = 20$$

IF 2 appears two times:

$$\textcircled{1} \dots 2, 2, \boxed{a}, \boxed{b}, 8$$

$$\boxed{1}, 2, 2, \boxed{7}, \boxed{7} \leftarrow \text{Not correct because sum must be 20.}$$

$$\boxed{1}, \boxed{1}, 2, 2, \boxed{4} \leftarrow \text{Not correct because 1 should NOT appear two times.}$$

$$\boxed{1}, \boxed{1}, \boxed{1}, 2, 2 \leftarrow \text{Not correct because range must be 6.}$$

$$\textcircled{1} a + b = 8 \quad 2, 2, \boxed{a}, \boxed{b}, 8$$

$$3 \quad 5$$

$$2 \quad 6$$

$$4 \quad 4$$

\leftarrow 4 should not appear two times as mode = 2.

Ans: 2, 2, 3, 5, 8 or

2, 2, 2, 6, 8

8. 5 numbers, mean = 7

$$\text{Sum of the five numbers} = 7 \times 5 = 35$$

IF 7 appears two times:

$$7, 7, \boxed{a}, \boxed{b}, \boxed{17} \leftarrow \text{Not correct because } a + b = 4 \text{ and it can't be less than 7.}$$

$$\textcircled{1} \dots \boxed{a}, 7, 7, \boxed{b}, \boxed{c}$$

$$\textcircled{1} \dots \boxed{a}, \boxed{b}, 7, 7, \boxed{c}$$

$$\boxed{1}, \boxed{1}, \boxed{1}, 7, 7 \leftarrow \text{Not correct because range must be 10.}$$

$$\textcircled{1} a + b + c = 21$$

$$c - a = 10 \quad c = a + 10$$

$$a + b + a + 10 = 21$$

$$2a + b = 21 - 10$$

$$2a + b = 11$$

a	b	2a+b	c
1	9	11	11
2	7	11	12
3	5	11	13

Ans: 1, 7, 7, 9, 11

2, 7, 7, 7, 12

3, 5, 7, 7, 13