About Average: Stage 3* Zach

I used the formula: mean x number of quantities = sum of quantities, to find a target figure for each set.

1) Mean of 3 for 3 numbers, 3x3 = 9 (sum of quantities for this set)
Because 2 is the mode, it must appear at least twice: 2 + 2 + n = 9, so n = 5 Set is 2,2,5

2) Mean of 7 for 3 numbers, 7x3 = 21 (sum of quantities for this set)
Mode of 10, so 10 must appear at least twice: 10 + 10 + n = 21, so n = 1 Set is 10,10,1

3) Mean of 8 for 3 numbers, 8x3 = 24 (sum of quantities for this set)
Median of 10, so 10 is the second quantity: a + 10 + b = 24, so a + b = 14.
Range = 8, there are 7 possible ways to make 14 using positive integers, but only (3,11) has a range of 8.
Set is 3,10,11

4) Mean of 7.5 for 4 numbers, 7.5x4 = 30 (sum of integers in this set)
Mode is 6, so 6 must appear at least twice, so 6 + 6 + a + b = 30. To calculate a median, you have to list the numbers in order of size. The median is 7, and there is an even number of quantities in this set, so 7 is also the mean of the middle two numbers: 6 + a = 14 (14÷2 = median 7), so a = 8 and b = 10. Set is 6,6,8,10

5. Mean of 6 for 4 numbers, 6x4 = 24 (sum of integers in this set)

a + **c** + **d** + **b** = 24

If the median is 6.5, and there is an even number of integers in the set, then 6.5 is also the mean of the middle two quantities: **c + d = 13 (13÷2 = 6.5).** There are 6 possible pairs of positive integers that add up to 13: 1+12, 2+11, 3+10, 4+9, 5+8 and 6+7; and these must lie in the middle of the set so that a<**c**<**d**<b

a + c + d + b = 24, **but if c + d = 13**, **then a +b must = 11.** (13 + 11 = 24)

<u>But this is not possible using only positive integers.</u> The smallest positive integer is 1; zero is neither negative nor positive. You can solve this using non-negative integers (includes zero): {0,1,2,3,4,5, ... }, but this is NOT the same as using positive integers: {1,2,3,4,5, ... }, which is what the question asks for.

Proof:
0,6,7,11 satisfies: median 6.5, range 11, mean 6,
1,6,7,12 satisfies: median 6.5, range 11, but mean 6.5 (26÷4 = 6.5)
2,6,7,13 satisfies: median 6.5, range 11, but mean 7 (28÷4 = 7)

In this model, you are adding 2 to the sum of integers every time, so the mean increases by 0.5.

6) Mean of 4 for 5 numbers, 4x5 = 20 (sum of integers in this set)
Mode is 3, so 3 must appear at least twice. Range = 9. The possible pairs of non-negative integers that satisfy a range of 9 are: (0,9), (1,10), (2,11) etc. I used trial and error, to find a solution.

0	+	3	+	3	+	n	+	9	=	20	n = 5: {0,3,3,5,9} satisfies mean = 4, mode = 3, range = 9, but uses a non-negative integer (zero is neither negative nor positive)
1	+	3	+	3	+	n	+	10	=	20	n = 3: {1,3,3,3,10} satisfies mean = 4, mode = 3, range = 9,
2	+	3	+	3	+	n	+	11	=	20	n = 1 {1,2,3,3,11} satisfies mean = 4, mode = 3, but range = 10

As in question 5, you are adding 2 to the sum of integers as you progress down the table, but this time can use n as a balancing figure (which decreases by 2, to compensate) and so maintain a mean of 20.

Using positive integers only: 1,3,3,3,10 works.
Using non-negative integers, then 0,3,3,5,9 also works (but understand that zero is neither positive nor negative).

7) Mean of 4 for 5 numbers, 4x5 = 20 (sum of integers in this set). Mode is 2, so 2 must appear at least twice. Range = 6, pairs of possible positive integers with range 6 are: (1,7), (2,8), (3,9) etc.. Using trial and error I found that only the range pair (2,8) worked, and then used this to find the second pair:

$$2 + 2 + a + b + 8 = 20 \qquad \text{so } a + b = 8$$

2,2,2,6,8 and 2,2,3,5,8 satisfy the criteria.

8. Mean of 7 for 5 numbers, 7x5 = 35 (sum of integers in this set). Mode is 7, so 7 must appear at least twice. The three numbers left total 21 (35 – 7 – 7 = 21). Range = 10. To find all three sets, start at 1, and increase the first quantity, whilst maintaining a range of 10. Then use a **balancing figure** to maintain a mean of 7.

Sets are:
1,7,7,**9**,11
2,7,**7**,7,12
3,**5**,7,7,13

Extension

Find sets of positive integers with mean 4, mode 1, median 2 and range 10
a) Four numbers = 1,1,3,11
b) Five numbers = 1,1,2,5,11
c) Six numbers = 1,1,1,3,7,11
d) One hundred numbers – see spreadsheet for one possible solution.

I explored patterns on a spreadsheet for multiples of 10 integers, although once you get past six quantities, there are actually many possible solutions.