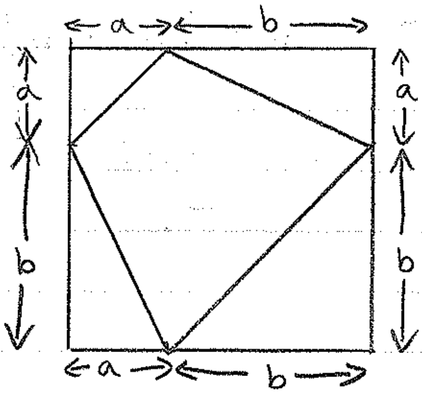
Quadrilaterals in a square



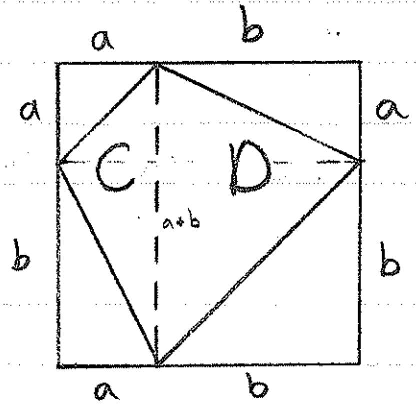
Qs) Prove that the area of the red quadrilateral is half of the yellow square.

* Arithmetical proof

→ The area of the yellow square is given by (base × height)

In this square therefore it is:
$(a+b) \times (a+b)$

→ The small * quadrilateral can be divided into two smaller triangles as shown →



Area of triangle C
$= \dfrac{a(a+b)}{2}$

Area of triangle D
$= \dfrac{b(a+b)}{2}$

Following the question, half the area of the yellow square will equal the two small triangles:

So:

$\frac{1}{2}$ Area yellow square = Area of small triangles

$$\frac{1}{2}(a+b)(a+b) = \frac{a(a+b)}{2} + \frac{b(a+b)}{2}$$

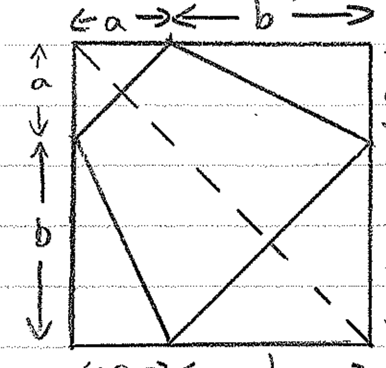
$$\frac{(a+b)(a+b)}{2} = \frac{a(a+b) + b(a+b)}{2}$$

Cancell out the fractions

$$(a+b)(a+b) = a(a+b) \times b(a+b)$$
$$(a+b)(a+b) = a^2 + ab + ba + b^2$$
$$(a+b)(a+b) = a^2 + 2ab + b^2$$
$$(a+b)(a+b) = (a+b)(a+b)$$

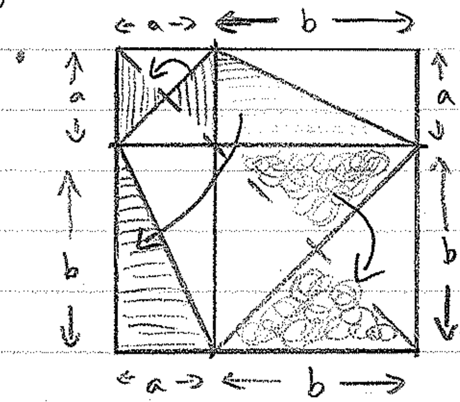
Proving the initial statement.

*Geometrical proof



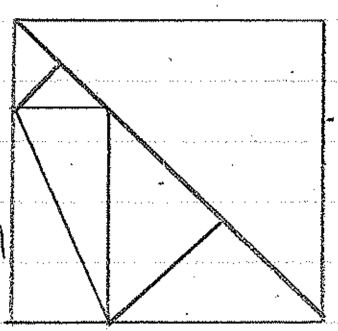
If we divide the square into two by drawing the diagonal, we get two equal right-angles triangles as shown

We then divide the quadrilaterals C and D, just like it was done before



The quadrilateral is now divided into four smaller quadrilaterals

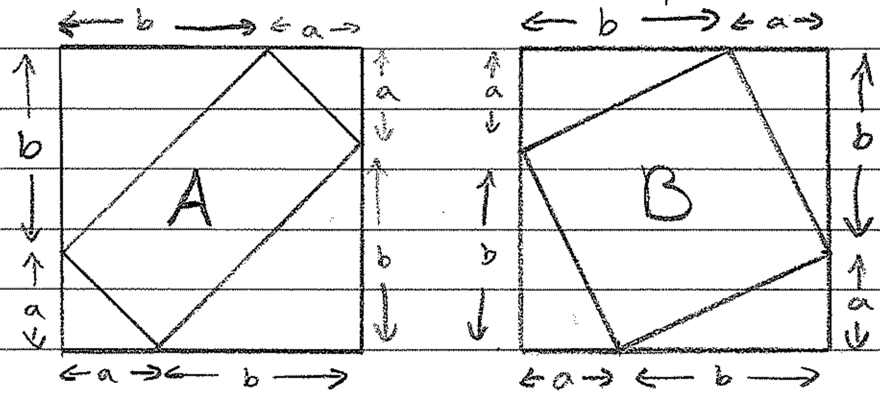
We can then rearrange the pieces so that they all fit inside one of the big right angled triangles



As only that triangle is covered, only half of the area is covered.

The area of the small quadrilateral is exactly half the area of the big quadrilateral.

Quadrilaterals in a square



Qs) Prove the area ~~that~~ of both red squares added, add to one big yellow square.

* Arithmetical proof

As before, the area of the big square is

$(a+b)(a+b) = a^2 + 2ab + b^2$

The area of shape A is:

$$\begin{aligned} A &= \text{base} \times \text{height} \\ &= \sqrt{a^2+a^2} \times \sqrt{b^2+b^2} \\ &= \sqrt{2a^2} \times \sqrt{2b^2} \\ &= \sqrt{4a^2b^2} \\ &= 2ab \end{aligned}$$

The area of shape B is:

$$\begin{aligned} A &= \text{base} \times \text{height} \\ &= \sqrt{a^2+b^2} \times \sqrt{a^2+b^2} \\ &= \left(\sqrt{a^2+b^2}\right)^2 \\ &= a^2+b^2 \end{aligned}$$

Therefore Area of shape A + area shape B

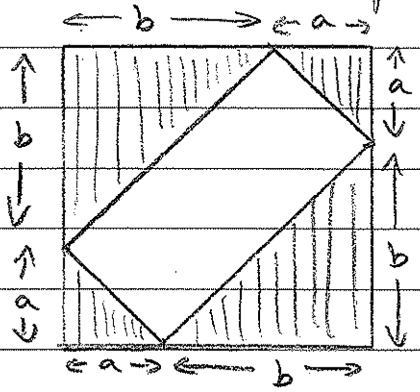
$= 2ab + a^2 + b^2$

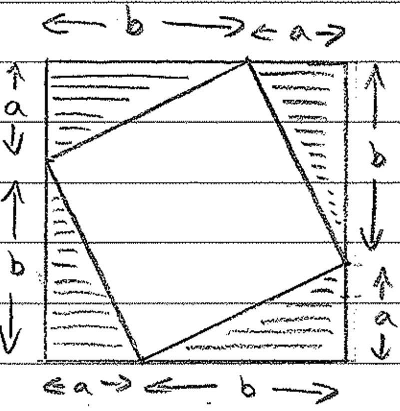
$= a^2 + 2ab + b^2$

Which is also the same as the big square area

Proving the initial statement

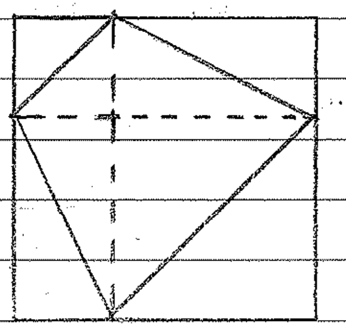
* Geometrical proof

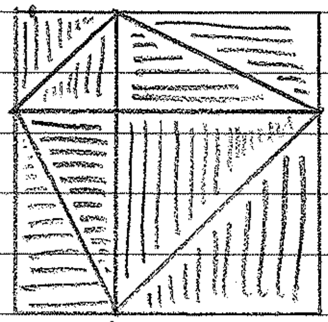




If the areas of both small quadrilaterals add up to one big square, then the remaining bits must also add to the big square.

If we take the quadrilateral from earlier on, we can fit in the remaining bits on to this square.





This shows and proves that the ~~th~~ two quadrilaterals from above add up to the big square

(Their areas are the same)