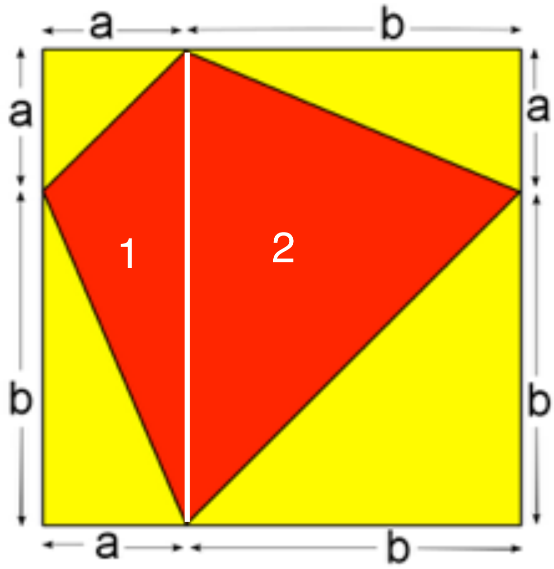
QUADRILATERALS IN A SQUARE

by: Katie Yoon (Dulwich College Seoul)

1. Can you prove that in each of these images the area of the red quadrilateral is exactly half the area of the yellow square?



ALGEBRAIC PROOF:

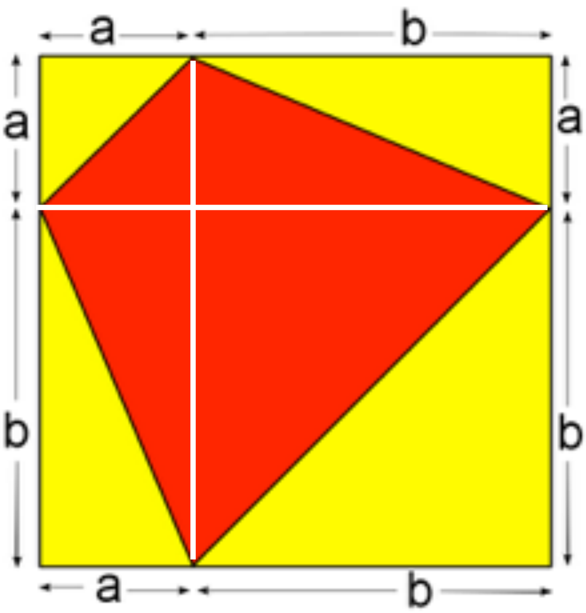
Area of the yellow square
= (a+b)(a+b)
= $a^2 + 2ab + b^2$

Area of red quadrilateral
In order to find the area of the red quadrilateral you must divide it into two triangles as shown on the right.

<u>formula of the area of a triangle = (b x h)/2</u>
1. Therefore, area of triangle 1
= a(a+b)/2
= $(a^2 + ab)/2$
2. area of triangle 2
= b(a+b)/2
= $(ab + b^2)/2$

Hence, the area of red quadrilateral
= $(a^2 + ab+ab + b^2)/2$
= $(a^2 + 2ab + b^2)/2$

<u>$(a^2 + 2ab + b^2)/2$ is half of $a^2 + 2ab + b^2$.</u>



GEOMETRIC PROOF:

Divide the red quadrilateral like as seen in the diagram on the left.

Now there are four quadrilaterals.

Each quadrilateral is divided into two congruent triangles.

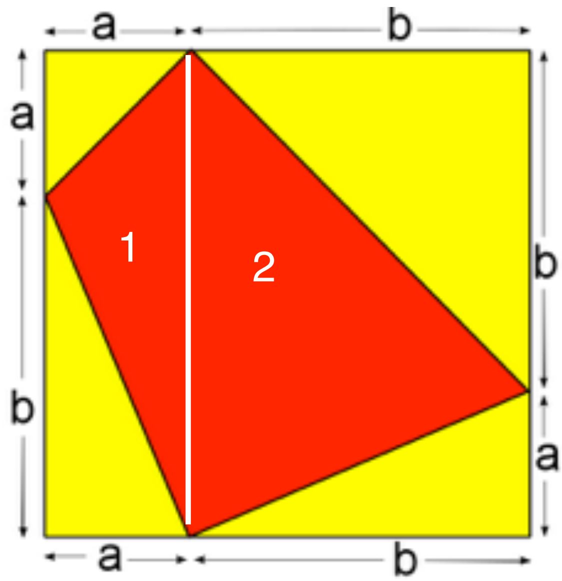
This means that the red part and the yellow part are equal.

This also means that the red quadrilateral is half the size of the yellow square.

$a^2/2 + b^2/2 + ab/2 + ab/2$
= $(a^2 + b^2 + ab + ab)/2$
= $(a^2 + 2ab + b^2)/2$

<u>$(a^2 + 2ab + b^2)/2$ is half of $a^2 + 2ab + b^2$.</u>

You can prove the other image utilizing similar methods.



ALGEBRAIC PROOF:

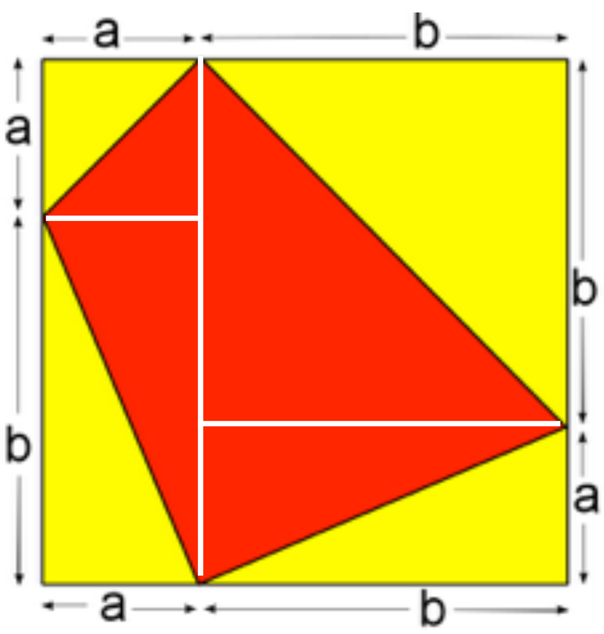
Area of the yellow square:
= (a+b)(a+b)
= $a^2 + 2ab + b^2$

Area of red quadrilateral:
divide red quadrilateral into two triangles as shown on the left.

<u>formula of the area of a triangle = (b x h)/2</u>
1) Therefore, area of triangle 1
= a(a+b)/2
= $(a^2 + ab)/2$
2) area of triangle 2
= b(a+b)/2
= $(ab + b^2)/2$

Hence, the area of the red quadrilateral
= $(a^2 + ab+ab + b^2)/2$
= $(a^2 + 2ab + b^2)/2$

<u>$(a^2 + 2ab + b^2)/2$ is half of $a^2 + 2ab + b^2$.</u>



GEOMETRIC PROOF:

Divide the red quadrilateral like as seen in the diagram on the left.

Now there are four quadrilaterals.

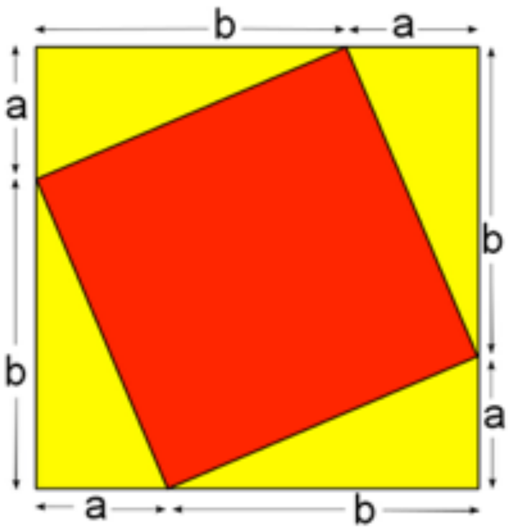
Each quadrilateral is divided into two congruent triangles.

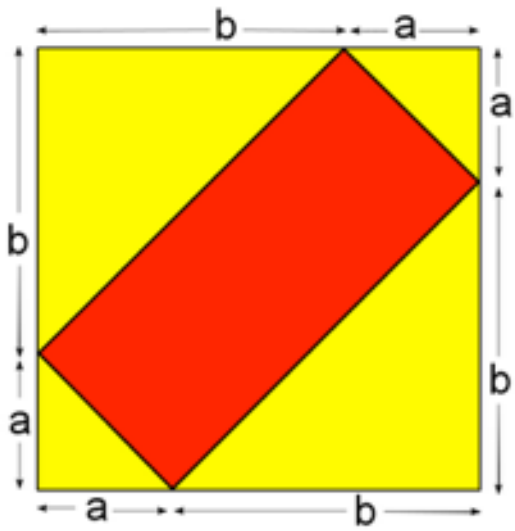
This means that the red part and the yellow part are equal.

This also means that the red quadrilateral is half the size of the yellow square.

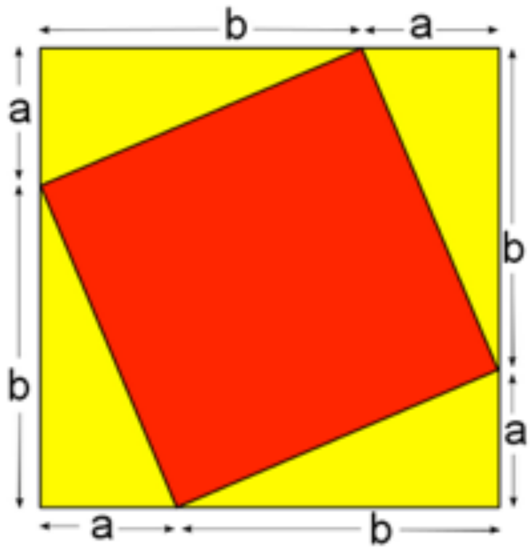
In order to prove that the areas of two red quadrilaterals above sum to the area of the yellow square, we can use the similar methods as before.

Can you prove that the areas of these two red quadrilaterals sum to the area of the yellow square?





ALGEBRAIC PROOF:

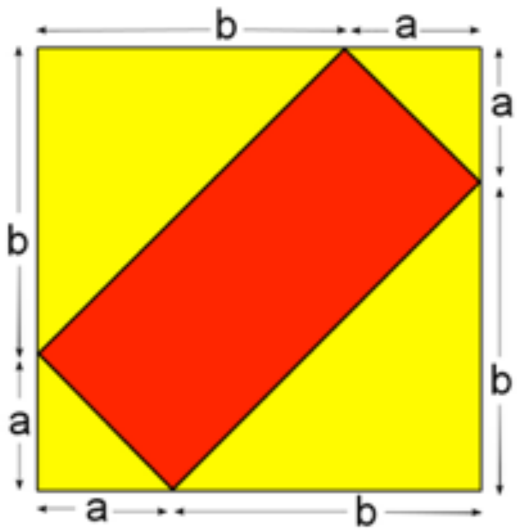


Area of the yellow square:
= (a+b)(a+b)
= $a^2 + 2ab + b^2$

In order to find the area of the red quadrilateral, you must first find the area of the yellow triangles and subtract it from the area of the yellow square.

Area of yellow triangles together
= (ab+ab+ab+ab)/2
= 4ab/2
= 2ab

Hence, the area of the red quadrilateral
= $(a^2 + 2ab + b^2) - 2ab$
= $a^2 + b^2$



Area of the yellow square:
= (a+b)(a+b)
= $a^2 + 2ab + b^2$

In order to find the area of the red quadrilateral, you must first find the area of the yellow triangles and subtract it from the area of the yellow square.

area of yellow triangles together
= $(a^2+b^2+a^2+b^2)/2$
= $(2a^2+2b^2)/2$
= $(a^2+b^2)/2$

Therefore, the area of the red quadrilateral
$= (a^2 + 2ab + b^2) - (a^2+b^2)/2$
$= (2a^2 + 4ab + 2b^2)/2 - (a^2+b^2)/2$
$= (2a^2 + 4ab + 2b^2 - a^2 - b^2)/2$
$= (a^2 + 4ab + b^2)/2$

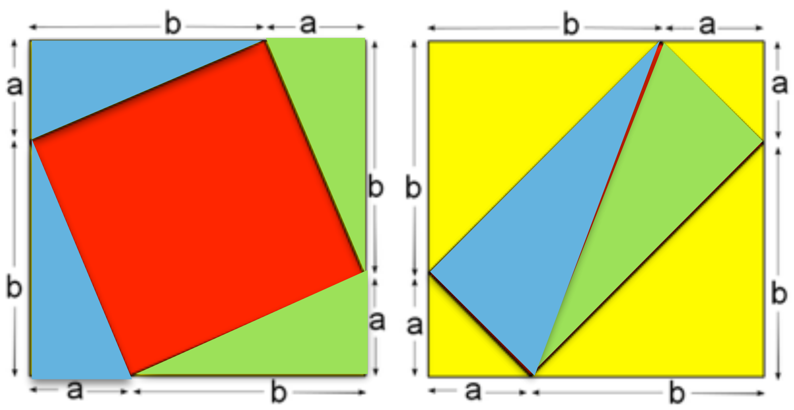
To find if the two red quadrilaterals equal to the area of the yellow square, you must add the area of the two red quadrilaterals together.
Sum of red quadrilaterals

$= [(a^2 + 4ab + b^2)/2] + [(a^2+b^2)/2]$
$= (a^2 + 4ab + b^2 + a^2+b^2)/2$
$= (2a^2 + 4ab + 2b^2)/2$
$= a^2 + 2ab + b^2$

GEOMETRIC PROOF:

As evident from the images above, if the sum of the red quadrilaterals are equal to a yellow square, then the yellow part of the left image is equal to the red quadrilateral on right, and the yellow part of the right image is equal to the red quadrilateral on the left.

You can show this as a diagram:



blue triangles on left
$= (ab + ab)/2$
$= 2ab/2$
= ab

blue triangle on right
$= (\sqrt{2a^2} \times \sqrt{2b^2})/2$
$= \sqrt{(2a^2 \times 2b^2)}/2$
$= \sqrt{4a^2b^2}/2$
$= 2ab/2$
= ab

<u>ab = ab</u>

This is the same for the green triangles:

green triangles on left
$= (ab + ab)/2$
$= 2ab/2$
= ab

green triangle on right
$= (\sqrt{2a^2} \times \sqrt{2b^2})/2$
$= \sqrt{(2a^2 \times 2b^2)}/2$
$= \sqrt{4a^2b^2}/2$
$= 2ab/2$
= ab

<u>ab = ab</u>