PUSHTOON NAZARI

1 Absolutely – Stage 5 – Solution

The graph is an absolute function:

$$f(x) = |x| \qquad x \in \mathbb{R}$$

The ~~graph~~ function has a number of interesting features such as:

- It is an even function as we can see the graph is symmetrical about the y axis or the line $x = 0$ which means it has the property $f(x) = f(-x)$ for all x where $x \in \mathbb{R}$.

- Furthermore the function is a proper function as every input has no more than one output. As the range of the function is $[0, \infty)$ and both domains $[0, \infty)$ and $(0, -\infty)$ contain linear functions

- This brings us to another feature of the function which is that it is a piecewise function. So we can express the function in two parts:

$$f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

The graph also demonstrates that as you move away from 0 there is a common change of 1 unit, ~~also the gradient at any point is |m|~~.

The absolute function may be used to calculate the distance between two points, or in the case of the graph given, the distance of a point from (0,0). Therefore absolute functions may be used to answer questions of "how far...?" such as when travelling. It is important to note that the absolute function will not take into account direction for instance if I start at home and set out on a journey 5 miles east from my home then 4 miles west, an absolute function will not show the 4 miles west as -4 but as $|-4| = 4$ as it is only a measurement of ~~distance from where I started~~ how far.

(2)

To move the 'corner' at the point $(0,0)$, will depend on what direction one wants to move. Below I have given the different functions for various types of shifts to move the 'corner' (vertex):

Horizontal shift to the right:

where $f(x) = |x|$

$g(x) = f(x-k)$ where $k>0$ and represents the number of units to the right to move the corner.

Horizontal shift to the left:

where $f(x) = |x|$

$h(x) = f(x+k)$ where $k>0$

Vertical shift upwards:

where $f(x) = |x|$

$m(x) = f(x) + k$ $k>0$

Vertical shift downwards:

where $f(x) = |x|$

$n(x) = f(x) - k$ $k>0$

The function looks similar to the function $f(x) = x$, however the difference being that the function has been reflected along the line $y=0$ for $(0, -\infty)$. ~~The difference between $f(x) = |x|$ and $f(x) = x$ property wise would be that $f(x) = x$ is not a proper function~~, it is at the $(0,0)$ where $f(x) = |x|$ with it's sharp $90°$ vertex moves away from the linear function of $f(x) = x$ as well as $f(x) = -x$.

(3)

Also $f(x) = |x|$ reminds me of the quadratic function $f(x) = x^2$ due to the similar symmetry about the y axis as well as their common features of being even & proper functions. I would like to know if there are further similarities between the functions & the differences?

Further questions I might ask is what would the equation be if it was reflected in the x axis and what could it model?

Also the vertex of $f(x) = |x|$ is a right angle could there be any link to pythagoras theorem, could we use modulus function to plot triangles using the x / y axis as the 3rd side. Or even when constraining the domain finding the maximum distance between points on the function $f(x) = |x|$; how could this be helpful?