

Rugby School

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Proving the laws of logarithms

Prove: $\log_c a + \log_c b = \log_c ab$

Let $a = c^x$ and $b = c^y$

$$\therefore \text{RHS} = \log_c c^{x+y}$$

$$= x+y$$

$$\& \text{LHS} = \log_c c^x + \log_c c^y$$

$$= x+y$$

$$\therefore \text{LHS} = \text{RHS}$$

QED

Prove: $\log_c a^b = b \log_c a$

Let $a = c^x$

$$\therefore \text{RHS} = b \log_c c^x$$

$$= bx$$

$$\& \text{LHS} = \log_c (c^x)^b$$

$$= \log_c c^{xb}$$

$$= xb$$

$$\therefore \text{LHS} = \text{RHS}$$

Prove $(\log_b a) \times (\log_c b) = \log_c a$

Let $a = b^x$

$$\therefore \text{RHS} = \log_c b^x$$

$$= x \log_c b$$

$$\& \text{LHS} = \log_b b^x \times \log_c b$$

$$= x \log_c b$$

$$\therefore \text{LHS} = \text{RHS}$$