(1)

We will prove that $\log_c a + \log_c b = \log_c ab$ for any $a, b > 0$ and $c > 0$, but $c \neq 1$.

(2)

As we don't know how to manipulate logarithms, but we do know how to manipulate powers, it might be helpful to turn expressions involving logarithms into expressions involving powers. To do this, it might be helpful to name $\log_c a$ and $\log_c b$.

(3)

Let $\log_c a = x$ and $\log_c b = y$.

(4)

We can rewrite these equations to give us two equations involving powers.

(5)

$c^x = a$ and $c^y = b$

(6)

We would like to express $\log_c ab$ in terms of $\log_c a$ and $\log_c b$. It may be helpful to express ab in terms of the base of the logarithm, c, since then we might be able to say more about $\log_c ab$.

(7)

$ab = c^x c^y$

(8)

Therefore $ab = c^{x+y}$

(9)

Therefore $\log_c ab = \log_c a + \log_c b$, as required.