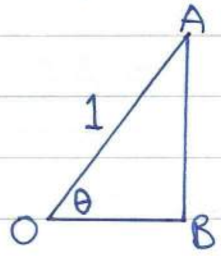
Find the following lengths in terms of $\sin\theta$ and $\cos\theta$

OB:



$\cos\theta = \frac{OB}{1}$

$= \boxed{OB = \cos\theta}$

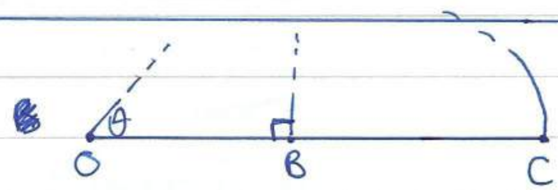
$\boxed{\text{SOH CAH TOA}}$

AB: $\sin\theta = \frac{AB}{1}$

$= \boxed{AB = \sin\theta}$

$\boxed{\text{SOH CAH TOA}}$

BC:



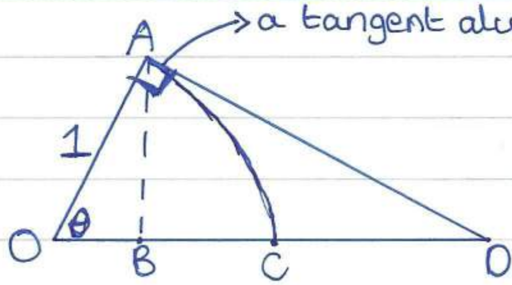
OC = 1, since the radius of the circle is 1

$OB = \cos\theta$

$BC = OC - OB = 1 - \cos\theta$

$\boxed{BC = 1 - \cos\theta}$ $\qquad$ $\text{versin}\,\theta = 1 - \cos\theta$

OD:



a tangent always meets the radius at right-angles

$\cos\theta = \frac{1}{OD}$

$= \boxed{OD = \frac{1}{\cos\theta}}$

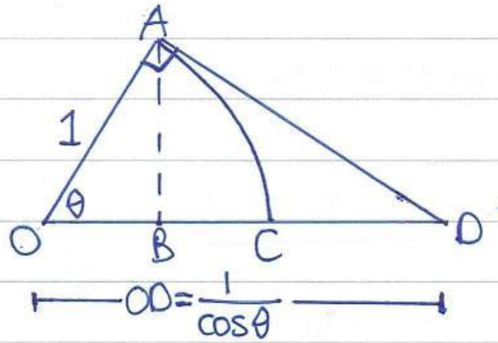
$\boxed{\text{SOH CAH TOA}}$

CD: $CD = OD - OC$

$= CD = \frac{1}{\cos\theta} - 1$

$= \boxed{CD = \frac{1-\cos\theta}{\cos\theta}}$ $\qquad$ $\text{exsec}\,\theta = \frac{1-\cos\theta}{\cos\theta}$

AD:

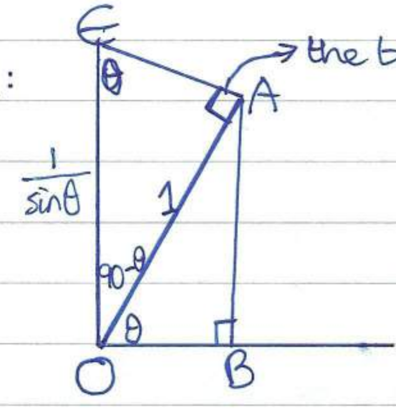


$$\sin\theta = \frac{AD}{\frac{1}{\cos\theta}}$$

$$= \sin\theta \times \frac{1}{\cos\theta} = AD$$

$$\Rightarrow \boxed{AD = \frac{\sin\theta}{\cos\theta}}$$

AE:



the tangent always meets the radius at 90°

Since $E\hat{O}B$ is a right-angle,
$E\hat{O}A = 90° - \theta$
Since angles in a triangle have a sum of 180°,
$O\hat{E}A = 180 - (90) - (90 - \theta)$
$\Rightarrow O\hat{E}A = \theta$

$$\therefore \sin\theta = \frac{1}{EO}$$

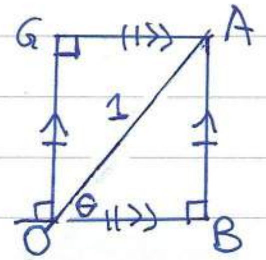
$$\Rightarrow \boxed{EO = \frac{1}{\sin\theta}}$$

SOH CAH TOA

$$\therefore \cos\theta = \frac{AE}{\frac{1}{\sin\theta}}$$

$$\boxed{AE = \frac{\cos\theta}{\sin\theta}}$$

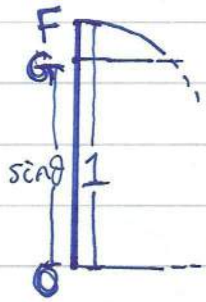
AG:



AG and OB are parallel, equal lines in a rectangle (since all angles in rectangle OGAB are parallel). Therefore,
$\boxed{AG = OB = \cos\theta}$

GO: GO = AB as they are parallel, equal lines in a rectangle. $\boxed{GO = AB = \sin\theta}$

FG:



$FO = 1$, since the radius is 1

$GO = \sin\theta$

$FG = FO - GO$

$= \boxed{FG = 1 - \sin\theta}$ $\quad$ $\text{coversin}\,\theta = 1 - \sin\theta$

EF:

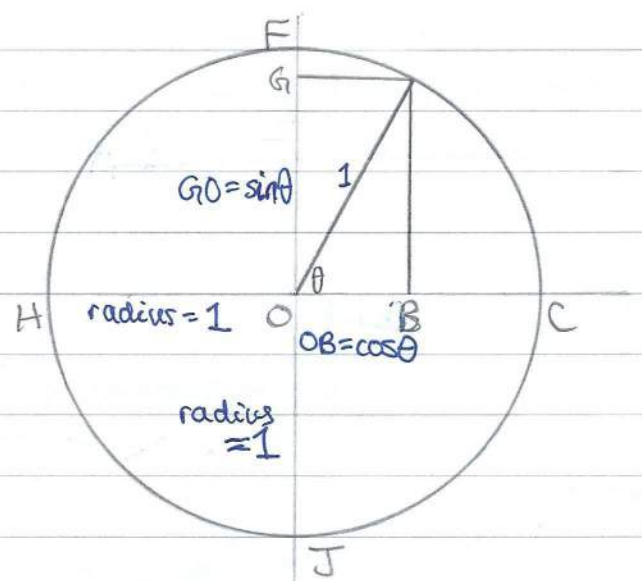
$$EF = EO - FG - GO$$

$$= \frac{1}{\sin\theta} - (1 - \sin\theta) - \sin\theta$$

$$= \frac{1}{\sin\theta} - 1 + \sin\theta - \sin\theta$$

$$= \frac{1}{\sin\theta} - 1$$

$$= \boxed{EF = \frac{1 - \sin\theta}{\sin\theta}}$$



$\text{vercos}\,\theta = 1 + \cos\theta$

$= HO + OB$

$= \boxed{\text{vercos}\,\theta = HB}$

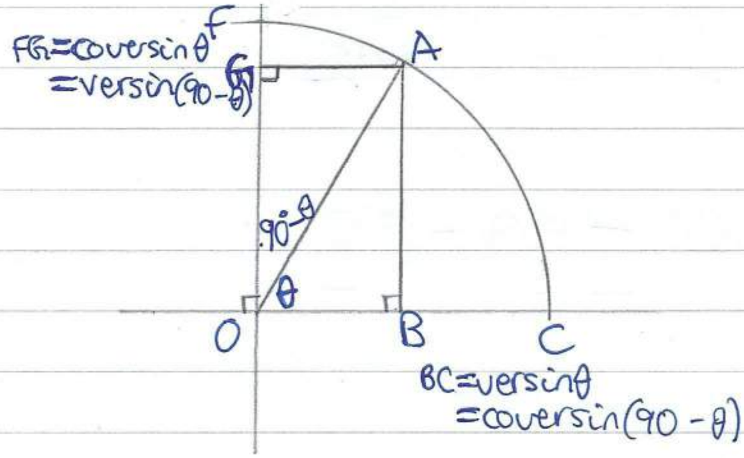
$\text{covercos}\,\theta = 1 + \sin\theta$

$= JO + OG$

$= \boxed{\text{covercos}\,\theta = JG}$

relationship between <u>versine</u> and <u>coversine</u>:

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$



$BC = \text{versin}\,\theta$
$FG = \text{coversin}\,\theta$
$G\hat{O}A = 90° - \theta$ because ~~since~~
$A\hat{O}B = \theta$

complimentary angle

$\text{coversin}\,\theta = FG$, so $\text{versin}(90° - \theta) = FG$ also.

$\therefore \text{coversin}\,\theta = \text{versin}(90° - \theta)$

$90° = \frac{\pi}{2}$ (radians)

$$\therefore \boxed{\text{coversin}\,\theta = \text{versin}\left(\frac{\pi}{2} - \theta\right)}$$

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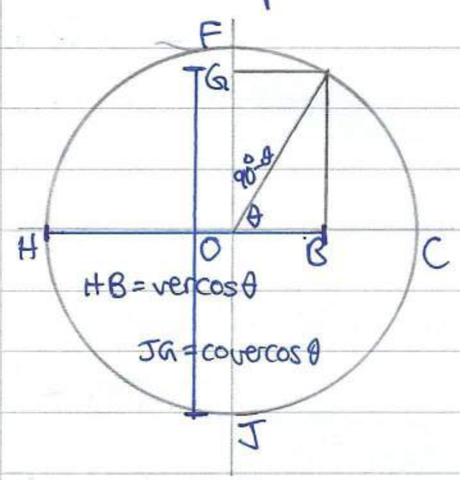
relationship between <u>vercosine</u> and <u>covercosine</u>:



$HB = \text{vercos}\,\theta$
$JG = \text{covercos}\,\theta$

complimentary angle

$JG = \text{vercos}(90° - \theta)$
$HB = \text{covercos}(90° - \theta)$

$\underline{\text{covercos}\,\theta = \text{vercos}(90° - \theta)}$

$$= \boxed{\text{covercos}\,\theta = \text{vercos}\left(\frac{\pi}{2} - \theta\right)}$$

The prefix "co" means to apply the function to the complimentary angle. (complimentary angles have a sum of 90°)