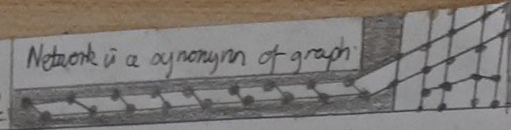
Graph Theory

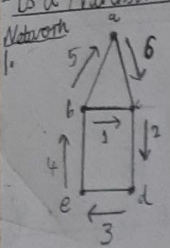
Can you Traverse It?

Network is a synonym of graph



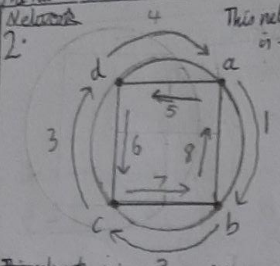
- Is it Traversible or Not? (Note - The numbers means the steps in order to traverse the network.)

Network 1.



This network is traversible.

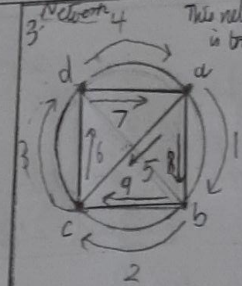
Network 2.



This network is traversible

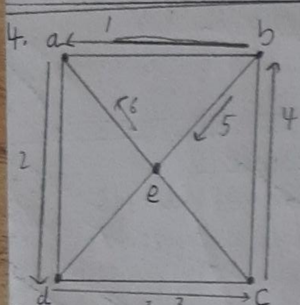
~~This network is not traversible~~

3. Network



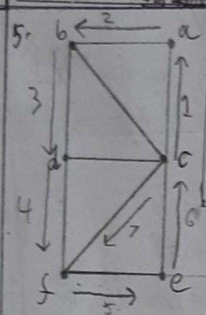
This network is traversible.

4.



This network is not traversible

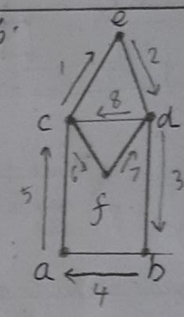
5.



This was just one of the many attempts to traverse the graph. For non-traversible graph, there will be one method to try traverse it.

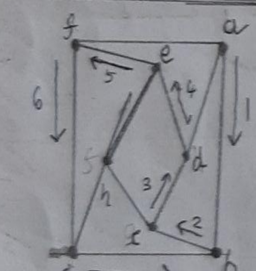
This network is not traversible

6.



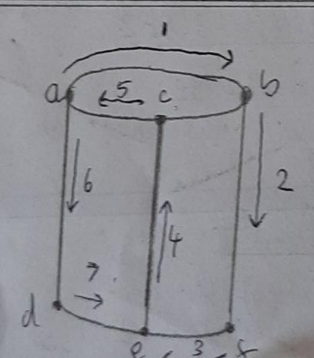
This graph was traversible.

7.



This graph is not traversible.

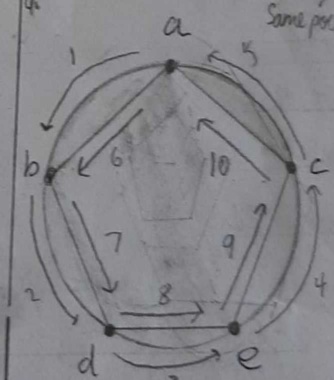
8.



This graph is not traversible

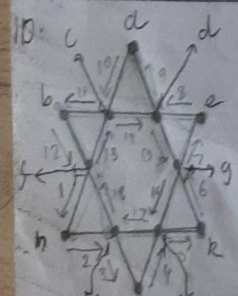
9.

Same points



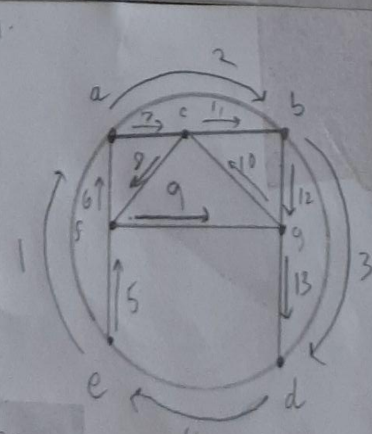
This graph is traversible

10.



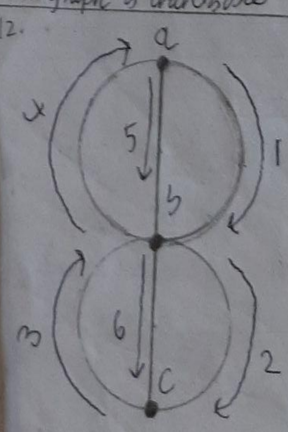
This graph is traversible

11.



This graph is traversible.

12.



This graph is traversible.

Findings and Conclusion + Observation Report

Note - 'Node' in the paper is another synonym for vertex.

Findings (Please see previous page to refer to the networks)

Network 1

Number of Nodes: 5

Number of steps taken to traverse: 6

Number of Edges: 6

Number of Edges connected to ~~each~~ node:

a-2 · b-3 · c-3 · d-2 · e-2

Traversible: Yes

Key

Network 2

Number of Nodes: 4

Number of steps ~~needed~~ to traverse taken: 8

Number of Edges: 8

Number of edges connected to node:

a-4 · b-4 · c-4 · d-4

Traversible?: Yes

Network 3

Number of Nodes: 4

Number of Edges: 9

Number of edges connected to ~~each~~ node: a-5, b-4, c-5, d-4

Traversible: Yes

Number of steps taken to traverse: 9

Network 4

Number of Nodes: 5

Number of Edges: 8

Number of edges connected to node:

a-3, b-3, c-3, d-3, e-4

Traversible: No

Maximum Number of edges traversed: 6

Network 5

Number of Nodes: 6

Number of Edges: 9

Number of Edges connected to node: ~~each~~ a-2, b-3, c-5, d-3, e-2, f-3

Traversible: No

Max no. of edges traversed: ~~7~~ 8

Network 6

Number of Nodes: 6

Number of Edges: 8

Number of Edges connected to node: a-2, b-2, c-3, d-3, e-2, f-2

Traversible: Yes

Number of steps taken to traverse: 8

Network 7

Number of Nodes: 8

Number of Edges: 12

Number of edges connected to node: a-3, b-3, c-3, d-3, e-3, f-3, g-3, h-3

Traversible - No

Maximum number of steps traversed: ~~8~~ 9

Network 8

Number of nodes - 6

Number of edges - 8

Number of edges attached to node: a-3, b-3, c-3, d-2, e-3, f-2

Traversible: No

Maximum number of steps traversed: 7

Network 9

Number of Nodes: 5

Number of Edges: 10

Number of Edges connected to node: a-4, b-4, c-4, d-4, e-4.

Traversible: Yes

Number of steps taken to traverse: 10

Network 10

Number of Nodes: 12

Number of Edges: 18

Number of Edges connected to nodes:

a, b, e, h, k, l - 2

d, c, i, j, f, g - 4

Traversible: Yes

Traversable - Yes

Number of steps taken to traverse: 18

Network 11

Number of Nodes: 7

Number of Edges: 13

Number of Edges connected to nodes:

~~a and d-4~~ a-4, b-4, c-4

~~c and b and~~ d-3, e-3, f-4

g-4

Traversable - Yes

Network 12

Number of Nodes: 3

Number of Edges: 4

Number of Edges connected to node: a-2, b-4, c-2

Traversible - Yes

Number of steps taken for traversion - 4

My Observations Report

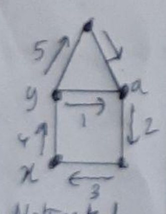
~~Hypothesises~~ Hypotheses on the Traversibility of a Network

While participating in providing a ~~solution theory~~ condition in order to determine whether a network was traversible or not. I was aware that the traversibility of a given network depended either on the number of nodes, edges or both. I already knew that size and curvature had no effect on the traversibility of a network (as long as no new nodes are created). Some of my hypotheses initially were that:

1. If a node had an odd-number of edges connected to it, the whole network was untraversible.
2. And in order for the network to be traversible, all the nodes should have an even-number of edges connected to it. } Odd or Even

I had believed that this was the basis for the traversibility of a network, however ~~there was~~ I was soon proven wrong by my very first network.

The Problem.



Network 1

Upon attempting to solve the problem of determining whether this network was traversible or not, I had started on Node X, which lead me to believe that this network was untraversible. However, during the next day, I was just practicing determining whether a graph was traversible or not. Out of boredom, I decided to determine the traversibility of the graph again, only this time, to start at the node now marked Y. To my astonishment, it was traversible (shown by the arrows). This led me to discard my "Odd or Even" Hypothesis (since the nodes y and a have 3 edges connecting to it) and also made me aware to the fact that repeated trial-and-error was a way of determining the traversibility of the graph, since the first test isn't always reliable.

My ~~Flee~~ New Hypotheses

1. Networks which all have an even-number of edges connected to each node are traversible.

My Observations on Traversable ~~and Non-Traversable~~ Networks

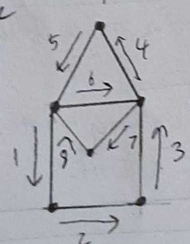
Out of the twelve networks I worked on, 8 of them were traversable. Through the exercise, I'd found out many remarkable ~~properties~~ characteristics of the networks which were traversable ~~or~~ and those that were not. However, I'd only used 12 graphs to find out about their properties, so some novel networks might be included in this part of the report for verification purposes.

Characteristics of Traversable Networks (Ending at the Starting Point)

Out of the 8 networks that were traversible, networks (2), (6), (9) and (10) all ended at the same point when traversed across. In all four of these graphs, one common method of traversing them was just to trace the outer edges of the graphs, and then trace the inside ~~part~~ edges of the networks. Furthermore, the nodes and the edges have an HCF larger than 1.

(Typically ~~shapes~~ polygons drawn in circles are usually ~~be~~ traversable, when you start and finish at the same place).

Example

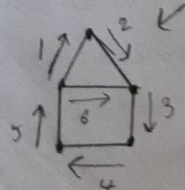


Network 6: Which has 6 nodes and 8 edges, the nodes and edges have an HCF of 2. ~~Ds~~ With the exception of Network 5, the difference between the edges and nodes of the network result in an even number.

Traversable Networks that End in Different Places:

The other 4 traversable networks that finish in different places, namely networks (1), (3), ~~[illegible]~~, (11) and (12). In all four of the networks, the nodes and edges do not have a common HCF other than 1. Furthermore, in all four of these graphs, the difference between the edges and the nodes of the network results in an odd number.

Example



Network 1, a traversable network where you start and finish at different places. It has 6 edges and 5 nodes, ~~th~~ subtract the edges by the nodes and you get 1, an odd number

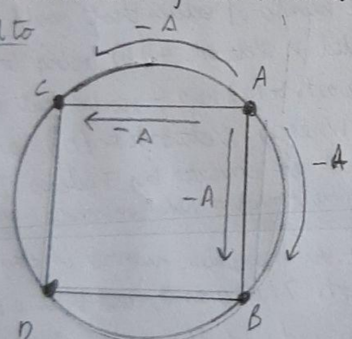
My Observation on Vertices ~~and Non-Traversable Networks~~

Note: In this paper, nodes will be used as a synonym for vertices.

Vertices in Traversable Graphs

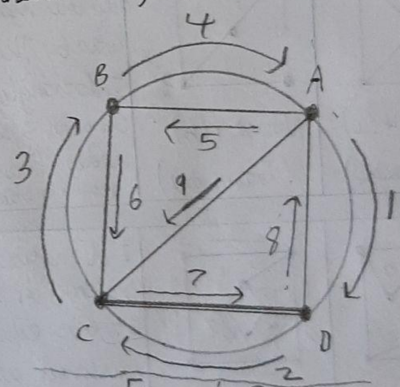
In the traversable graphs were you start and finish in different places, ~~each~~ at least one node is either connected ~~to a minimum~~ to a minimum of at least 3 other nodes, or to another node two times. And in the traversable graphs were you start and finish in ~~diff~~ the same place, you usually traversed them by traversing the outer part of the graph, then traversing the inner part. During this process, you visit the starting point again, and finish at the starting point once the inner part of the network is traced. Furthermore, in traversable networks that start and end in the same place, either each node is connected to two other nodes two times or that at least a minimum of two nodes are connected to four other nodes.

-A = A is connected to



Example 1

Network 2, a traversable network where you start and finish in the same place. All 4 nodes are connected to 2 other. This is demonstrated through Node A.



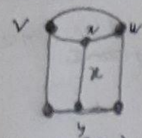
Example 2

Network 3, a traversable network where you start and finish at different places. Nodes B, C and D are connected to two other nodes two times. Node A however, is connected to an additional node C, ~~or~~ through an edge.

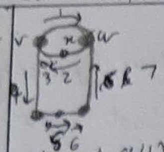
My Observations on Non-Traversable ~~Graphs~~ Networks

Non-Traversable Networks

Out of 12 networks I worked on, only 4 of them were non-traversable



Network 8(a)
(Non-traversable)



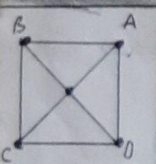
Network 8(b)
(Traversable after removing Edge x)

Network 8(a) was untraversable because the maximum number of edges that was possible to traverse was 7. However, by removing the edge marked x, the resulting network (Network 8(b)) was traversable as shown by the arrows.

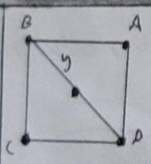
Network 8(a) – 6 Nodes, 8 Edges
Network 8(b) – 6 Nodes, 7 Edges

Previously node x was connected by 3 edges, as the same for node y. ~~As a result~~

In Network 8(a), nodes V, W, x and y were connected through three edges. In Network 8(b), only nodes ~~x~~ V and ~~y~~ W are connected through three edges. So, based on going over the previous networks, as long as a graph has only two nodes connected by an odd number of edges, it is still traversable.

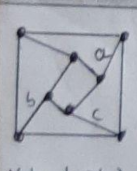


Network 4(a)
(Non-Traversable)

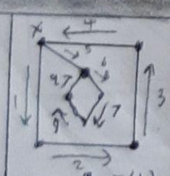


Network 4(b)
(Traversable)

I removed 2 edges out of Network 4 to make it traversable, as the maximum number of edges that could be traversed was 6. This resulted in Network 4(b) being traversable. Interestingly, Network 4(a) has 4 nodes that are connected through 3 edges. Whereas, Network 4(b) has only two nodes, B and A that are connected by 3 edges. Therefore, a graph is still traversible if only two nodes are connected by an odd number of edges.

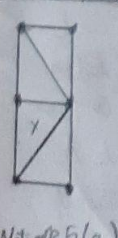


Network 7(a)
Non-Traversable

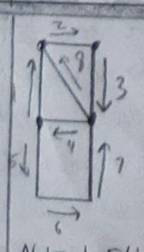


Network 7(b)
(Traversable)

I'd noticed that the maximum number of edges you could traverse in Network 7(a) was 9, and that you couldn't traverse edge a, b and c at the same time. So, I removed all three of them in order to create Network 7(b), another version of Network 7(a) that was traversable. Furthermore, Network 7(b) only has one node (x) that was connected by three edges.



Network 5(a)
Non-Traversable



Network 5(b)
Traversable

When Network 5(a) was untraversable, the maximum number of edges you could traverse was 8, with the removal of edge x, the resulting network (Network 5(b)) was traversable. (As shown by arrows).

Condition that determines the Traversability of a network:

Based on my observations determining the traversability of 12 networks. I, therefore state that in order for a graph to be traversable:-

1- All the nodes should have an even number of edges attached to it.

2- If not all nodes are even, at least two nodes can be connected by an odd number of edges.

Therefore, in order for a graph not to be traversable; it:

1- Should not have all ~~the~~ it's nodes connected by an even number of edges

2- It should have more than 2 of its nodes connected by an odd number of edges.

The reason these conditions determine the traversability of the graph is because, in order to be traversable, at least one node should be traversed twice. Additionally, the reason why networks with two nodes ~~are still traversable is because~~ that are connected by an odd number of edges is traversable is because of the fact that upto a maximum of two ~~od~~ nodes connected by an ~~even~~ odd number of edges can be connected in a network so that at least one point (node) is traversed twice. If the number of nodes connected by an odd number of edges (odd nodes) is larger than 2, this creates nodes in the network that are usually accessible by a few number of edges (usually 2 or 1). And since while traversing a graph, you can only cross 1 edge once, this limits the number of options you could choose to traverse ~~a point~~ to a point each time you traverse an edge until the network itself becomes untraversable.

Conclusion

In order for a graph to be traversable, at least one node will be traversed twice. Obviously, based on this condition, graphs with all of its ~~edg~~ nodes connected by an even number of edges are traversable. However, graphs with an upper bound of 2 nodes connected by an odd-number of edges are still traversable. The reason networks with ~~an odd~~ the number of nodes that are connected by an odd number of edges ~~is greater oth~~ is traversable ~~upto the number~~ as long as there are only two such nodes is because they can still be connected with other nodes with an even number of edges in a way that all the nodes are traversed, without one being traversed twice. If the number of nodes connected by an odd number of edges is greater than 3 or equal to it, this can ~~create~~ lead to certain nodes being accessible only by 1 or two paths, as the graph is being traversed, these nodes gradually become inaccessible as the paths are being used to traverse to other nodes, until the network itself becomes ~~untraversable~~ untraversable.