

Log Lattice

All logarithms are in the format of $\log_a b$

| | | Increasing Size | | | |
|-----------------|---|-----------------|------------|------------|------------|
| | | 2 | 3 | 4 | 5 |
| Increasing Size | 5 | $\log_5 2$ | $\log_5 3$ | $\log_5 4$ | 1 |
| | 4 | $\log_4 2$ | $\log_4 3$ | 1 | $\log_4 5$ |
| | 3 | $\log_3 2$ | 1 | $\log_3 4$ | $\log_3 5$ |
| | 2 | 1 | $\log_2 3$ | $\log_2 4$ | $\log_2 5$ |
| | | | | | |

All the logarithms to the top-left half of the table are smaller than 1. All the logarithms to the bottom-right half are bigger than 1.

The smallest logarithm is in the top-left corner ($\log_5 2$) while the largest ($\log_2 5$), is in the bottom-right corner.

So let $\log_5 2$ and $\log_2 5$ be fixed:

| | | Increasing Size | | |
|-----------------|------------|-----------------|------------|--|
| Increasing Size | $\log_5 2$ | | | |
| | | 1 | | |
| | | | $\log_2 5$ | |

The next two smallest logarithms are $\log_4 2$ and $\log_5 3$ while the next two largest ones are $\log_2 4$ and $\log_3 5$.

We can place them in as shown below:

| | | Increasing Size | | |
|-----------------|------------|-----------------|------------|--|
| Increasing Size | $\log_5 2$ | $\log_5 3$ | | |
| | $\log_4 2$ | 1 | $\log_3 5$ | |
| | | $\log_2 4$ | $\log_2 5$ | |

| | | Increasing Size | | |
|-----------------|------------|-----------------|------------|--|
| Increasing Size | $\log_5 2$ | $\log_4 2$ | | |
| | $\log_5 3$ | 1 | $\log_2 4$ | |
| | | $\log_3 5$ | $\log_2 5$ | |

The two logarithms from each set ($\log_4 2, \log_5 3$ and $\log_2 4$ and $\log_3 5$) can switch places.

Any two logarithms can be chosen from the remaining six and put into the two blank spaces as they would always fit the table.

Some possible answers:

| | | Increasing Size → | | |
|-------------------|------------|-------------------|------------|------------|
| | | $\log_5 2$ | $\log_5 3$ | $\log_3 4$ |
| Increasing Size ↓ | $\log_4 2$ | 1 | $\log_3 5$ | |
| | $\log_4 3$ | $\log_2 4$ | $\log_2 5$ | |
| | | | | |

| | | Increasing Size → | | |
|-------------------|------------|-------------------|------------|------------|
| | | $\log_5 2$ | $\log_4 2$ | $\log_5 3$ |
| Increasing Size ↓ | $\log_5 3$ | 1 | $\log_2 4$ | |
| | $\log_2 3$ | $\log_3 5$ | $\log_2 5$ | |
| | | | | |