Pocket Money

Imagine your family agreed to give you some pocket money every day for a month! You can choose from the following options:

- £10 every day
- £3 on the first day, £3.50 on the second, £4 on the third, and so on, increasing by 50p per day
- 1p on the first day, 2p on the second, 4p on the third, and so on, doubling each day.

Without doing any working out, which one would you choose, and why?

In a month with 31 days, how much money would you have by the end of the month, under each system? You may wish to explore using a spreadsheet...

Here are some questions you might like to consider:
In which months would option 1 be better than option 2?

February with 28 days. As the amount that you get is the same on the 29th day and gets overtaken by the 2nd opinion on the 30th day.

If your family stopped your pocket money on day 8, which option would give you the most?

On the 8th day the 1st option will have the most followed by the 2nd day then the 3rd day.

On which day of the month does option 3 become the most fruitful?
If you choose option 3, how many days would it be before you became a millionaire?

It will take 27 days before you became a millionaire.

The Solution

Without doing any working out, which one would you ***choose***, and ***why***?

As a group, without doing any working we believed that the 3rd option would be the best because of exponential growth after being given an example at the beginning of the year(Wheat and chessboard problem). We decided to take two approaches on this problem, they were to use graphs to explain and to use algebra to explain. We also used a table with the following terms till 31, this allowed us to closely analyse the numbers.

Algebraic approach

For the algebraic approach, we decide the best way to present our solution was to use the term to term sequence for the amount of money that would be received daily. When we found the term to term sequence, and it was not clear whether the 1st or 2nd option was the best. But we could easily tell that the 3rd option would be the best, since 2 is to the power of n which increases by the term number and is only divided by 200. For the other options we decided that it would be best if we were to use the graphs to compare.

Option 1:
£10 every day

Term to term rule: T(n) = 100

Option 2:
£3 on the first day, £3.50 on the second, £4 on the third, and so on, increasing by 50p per day

Term to term rule: T(n)= 0.5n+2.5

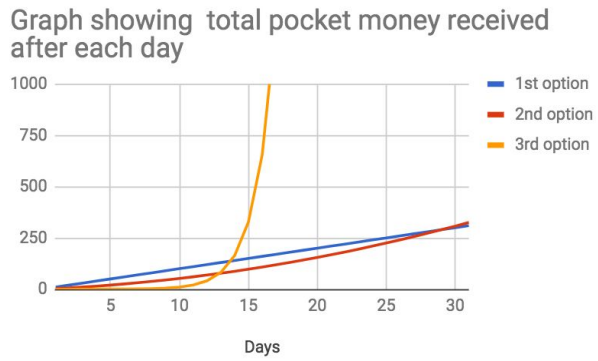
Option 3:
1p on the first day, 2p on the second, 4p on the third, and so on, doubling each day.

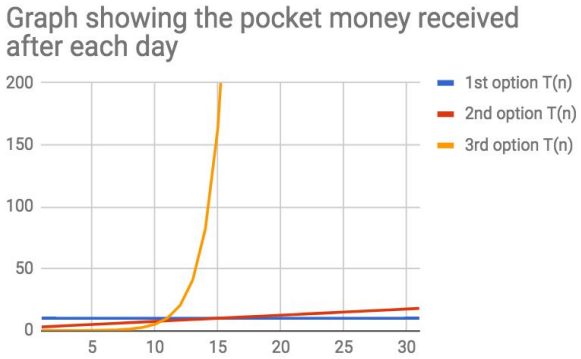
Term to term rule: T(n)= $\frac{2^n}{200}$

Graphical approach

Again with the graphs we can clearly see that the third option was by far the one that would provide the most amount of money. Finally with the graph, we are able to see that the 3rd option would just give more money. Comparing both graphs, there is an interesting difference in the lines for option 1 and 2.

The graph with the cumulative pocket money received after each day, the lines meet at around the 29th day. The graph with the amount of pocket money that is received after each day, the lines meet at around the 18th day. It is almost half the number of days, that it takes for the 2nd option to catch up with the 1st option. But it would not be exactly half as they start at different amounts.





Results

By using this table which had the terms of each option to the 31st day, with both the cumulative and the daily amount of money that would be received. This allowed our group to closely look at the numbers to see when exactly the amount given would change between options. We were also aided by conditional formatting, green meant it gave the most, red meant it gave the least amount of pocket money. When there was a change in colour in the columns, this meant on the graphs that the lines intersected each other.

Days	1st option	2nd option	3rd option	1st option T(n)	2nd option T(n)	3rd option T(n)
1	10	3.0	0.01	10	3.0	0.01
2	20	6.5	0.03	10	3.5	0.02
3	30	10.5	0.07	10	4.0	0.04
4	40	15.0	0.15	10	4.5	0.08
5	50	20.0	0.31	10	5.0	0.16
6	60	25.5	0.63	10	5.5	0.32
7	70	31.5	1.27	10	6.0	0.64
8	80	38.0	2.55	10	6.5	1.28
9	90	45.0	5.11	10	7.0	2.56
10	100	52.5	10.23	10	7.5	5.12
11	110	60.5	20.47	10	8.0	10.24
12	120	69.0	40.95	10	8.5	20.48
13	130	78.0	81.91	10	9.0	40.96
14	140	87.5	163.83	10	9.5	81.92
15	150	97.5	327.67	10	10.0	163.84
16	160	108.0	655.35	10	10.5	327.68
17	170	119.0	1310.71	10	11.0	655.36
18	180	130.5	2621.43	10	11.5	1310.72
19	190	142.5	5242.87	10	12.0	2621.44
20	200	155.0	10485.75	10	12.5	5242.88
21	210	168.0	20971.51	10	13.0	10485.76
22	220	181.5	41943.03	10	13.5	20971.52
23	230	195.5	83886.07	10	14.0	41943.04
24	240	210.0	167772.15	10	14.5	83886.08
25	250	225.0	335544.31	10	15.0	167772.16
26	260	240.5	671088.63	10	15.5	335544.32
27	270	256.5	1342177.27	10	16.0	671088.64
28	280	273.0	2684354.55	10	16.5	1342177.28
29	290	290.0	5368709.11	10	17.0	2684354.56
30	300	307.5	10737418.23	10	17.5	5368709.12
31	310	325.5	21474836.47	10	18.0	10737418.24

Procedure

For this problem, we were told how much money is given for the first three days and how the amount of the money changes after each day and our group found that we could either find how much money is given each day or find the total amount of money that would be given after each day (cumulative). We did both, since we could add the amount given from the previous day by the new amount to get the cumulative figures.

We found the term to term sequence for each option and by using Google Sheets we were also able to find the following values into a table. Which we used Google Sheets to help us form the graphs to better display, and to explain what we found. In addition we used conditional formatting, this provided our group with an easy view of which option gave the most amount on what day.

With all the values checked, we then proceeded to create the two graphs; the cumulative and the daily amount. We then used the graphs to compare each option for both cumulative and daily allowance. With all of this information we formed our answer.