



A Frosty Puddle

Frosty the snowman is made from two uniform spherical snowballs, of initial radii $2R$ and $3R$. The smaller (which is his head) stands on top of the larger.

As each snowball melts, its volume decreases at a rate which is directly proportional to its surface area, the constant of proportionality being the same for both snowballs. During melting each snowball remains spherical and uniform.

Let V and h denote Frosty's **total** volume and height at time t .

- Show that, for $2R < h \leq 10R$, $\frac{dV}{dh} = \frac{\pi}{8}(h^2 + 4R^2)$
- Derive the corresponding expression for $0 \leq h < 2R$
- Sketch $\frac{dV}{dh}$ as a function of h for $4R \geq h \geq 0$. Hence give a rough sketch of V as a function of h .



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