Sum of Odd Numbers

An odd number is one that can be expressed in the form of $2n + 1$, where n is any integer. For example, $7 = 2(3) + 1$. That is, the sequence of the first 10 positive odd numbers (starting from 1) is as follows:

1, 3, 5, 7, 9, 11, 13, 15, 17, 19.

The sum of the first n terms in the sequence above equals n^2. Below is a proof for that statement.

First, note that the sum of the first k natural numbers (from 1 to k) is $\frac{k(k+1)}{2}$.

Let S be the sum of the first n natural number:

$$S = 1 + 2 + 3 + \ldots + k\text{-}1 + k$$

$$2S = 1 + 2 + 3 + \ldots + k\text{-}1 + k$$

$$+ k + k\text{-}1 + \ldots + 3 + 2 + 1$$

Adding the corresponding numbers in each column we obtain:

$$2S = k(k+1)$$

Therefore:

$$S = \frac{k(k+1)}{2}$$

Proof:

The aforementioned sequence of the first n positive odd numbers can also be written as:

$$1 + 0, 1 + 2, 1 + 4, 1 + 6, \ldots, 1 + 2(n - 1)$$

Each term comprises a 1 added to the nonnegative multiples of 2.

Let: $O_n = (1 + 0) + (1 + 2) + (1 + 4) + (1 + 6) + \ldots + (1 + 2(n - 1))$

$$= 1 * n + 2 + 4 + 6 + \ldots + 2(n - 1)$$

Let: $E = 2 + 4 + 6 + \ldots + 2(n - 1)$

$$E = 2(1 + 2 + 3 + \ldots + n - 1)$$

Recall that the sum of the first k natural numbers is $\frac{k(k+1)}{2}$*:*

So: $E = 2\frac{n(n-1)}{2}$

$$= n(n-1)$$

$$= n^2 - n$$

Hence:

$$O_n = n + E$$

$$= n + n^2 - n$$

$$= n^2$$ *(proved)*