Impossible Sums – Proof 1

Cut out the statements and put them in order, to prove that powers of 2 cannot be written as the sum of two or more consecutive numbers.

Numbers of the form 2^n have no odd factors other than 1	A
Write the sequences above each other so the terms line up and add corresponding terms	B
If k is odd then $2a+k$ is odd and so the sum has an odd factor of $2a+k$. Since $a \geq 1$ and $k \geq 1$ we have $2a+k > 1$	C
Therefore $S = \frac{(k+1)(2a+k)}{2}$	D
Consider the consecutive numbers $a, (a+1), (a+2), \cdots, (a+k)$, with $a \geq 1$ and $k \geq 1$	E
Since any sum of consecutive numbers must have an odd factor greater than 1, it cannot be of the form 2^n	F
$2S = (2a+k) + (2a+k) + \cdots + (2a+k) + (2a+k)$	G
Therefore in every case the sum of the consecutive numbers has an odd factor which is greater than 1	H
Write down the sum of the numbers in descending order: $S = (a+k) + (a+k-1) + \cdots + (a+1) + a$	I
There are $k+1$ terms in $2S$, each of which is equal to $(2a+k)$ therefore $2S = (k+1)(2a+k)$	J
If k is even then $k+1$ is odd and so the sum has an odd factor of $k+1$. Since $k \geq 1$ we have $k+1 > 1$	K
Write down the sum of the numbers in ascending order: $S = a + (a+1) + (a+2) + \cdots + (a+k)$	L