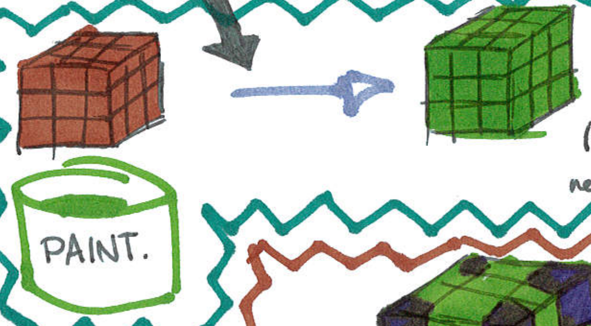
The Multilink Cube problem

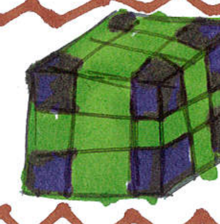
THE PROBLEM =

A cube made up of multilink cubes is dipped into a pot of paint. How many faces of the different cubes (0-6 faces) are covered in paint? Find the nth term formula for the number of faces painted on each cube from 0-6 with any sized cube.



ASSUMPTIONS =

- The paint doesn't go through any gaps in the full cube.
- For the number of cubes with 3 sides covered, the answer will always be 8 because there are always only 8 corner cubes.
- For any number of faces with more than four painted, the answer will always be 0 since the max faces showing is 3, UNLESS the cube is 1x1x1 because then it'd be 1 cube with 6 faces covered.

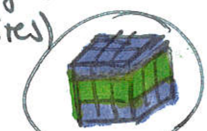


HOW TO WORK OUT THE FORMULA FOR 0 SIDES PAINTED

Firstly the obvious thing is that the cubes with 0 faces painted aren't visible, so you need to take off the outer layers.

(for a 3x3x3 there would be 1 cube not covered)

You do this by eliminating two of the outside layers (opposites)



Then, because it is cubed (^{3}) you need to cube the entire formula to get rid of the outer layer.

The formula must be = (n = width/height of the full cube)

$$(n-2)^3$$

HOW TO WORK OUT THE FORMULA FOR 1 SIDES PAINTED.

This is similar to the 0 sides painted, because again you have to take away the opposite layers to get the answer.

Then, instead of cubing it, if you look at the diagram =

you can see that the cubes are with 1 side painted are in a square formation. This proves that there won't be a cubed (^{3}) sign in the formula but a squared (^{2}) because they're squares.

Then, you need to multiply the whole thing by 6 because there are 6 sides on the cube and each has the square formation (shown above)

The formula must be =

$$6(n-2)^2$$

HOW TO WORK OUT THE FORMULA FOR 2 SIDES PAINTED =

(this uses a 3x3x3 cube as an example)

Firstly, you need $(n-2)$ in your formula because there are always two corners on each line of one face. → (purple)

This means

HOW TO SHOW IT WORKS=

obviously you can draw a table and count the squares and see that the formuli work.

no. of faces covered

length/width of cube	0	1	2	3	4	5	6	T*
1	0		0	0	0	0	1	1
2	0		0	8	0	0	0	8
3	1	6	12	8	0	0	0	27
4	8	24	24	8	0	0	0	64
n	$(n-2)^3$			8	0	0	0	n^3

exeption

$6(n-2)^2$ $12(n-2$

The total is always $=n^3$

*Total

However,

All the formuli add up to n^3 because that's always the total number of cubes.

$(n-2)^3=(n-2)(n-2)(n-2)$
$=(n^2+4-4n)(n-2$
$=n^3+4n-4n^2-2n^2-8+8n$ (crossed out terms)
$+6n^2+24-24n$
$+12n-24+8$

And if you test a random ones=

3x3x3 cube with 1 face covered is 6 ← table result

n=3

$6(3-2)^2$ → $3-2=1$ $1^2=1$

$1\times6=6$ → my result

4x4x4 cube with 2 faces covered is 24

n=4

$12(4-2)$ $12\times2=24$

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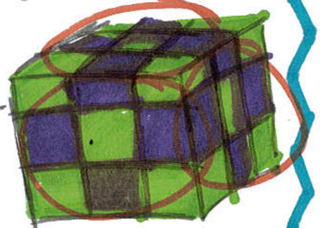
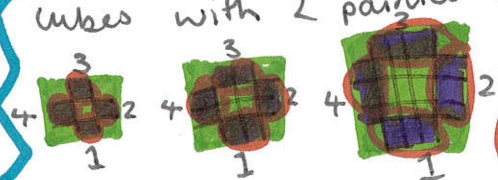
that you always have to take away 2 because the 2 corners are un-necessary and you don't count them because those cubes have 3 faces covered, not 2 which is what we are looking for.



So if you look at the diagram you can see that 4 cubes with 2 sides covered in paint on each face of the full cube, so if you times 4 by 6 (24) because there are 6 faces to the whole cube, but then you have to divide the answer by 2 because you're actually counting each cube twice because there are 2 sides painted. So you divide 24 by 2 (12). This doesn't only work for 3x3x3 as the 12 sides actually represent the area where the cubes with 2 painted faces are.



$4\times6=$
$24\div2=$
12

Now the formula must be

12(n-2)

because there are alway 12 Sections of the cube where the cubes with 2 faces painted can be, and then you take away 2 because there are always 2 corners in each column or row which you don't count.