Appendix 1
Progression of responses to Fair Feast

Establishing the nature of progression in fractions through children's responses to Fair Feast

As part of a NCETM Collaborative Project a group of teachers and subject leaders from Leicestershire used the Fair Feast rich task across a group of 6 schools. Each school used it with all the classes in the school (EYFS to Year 6) to varying degrees. The participants brought back evidence to the group to create this progression report.

Each 'key stage' has the expectations for the 2014 National Curriculum listed with it to analyse the extent to which the Fair Feast supports the teaching of the fractions objectives links directly to the National Curriculum objective possible additional links

'It has given me a good starting point with covering fractions over the year.' Year 5/6 teacher after using the fair feast with her class and asking the children adapt it using their own 'What if' questions

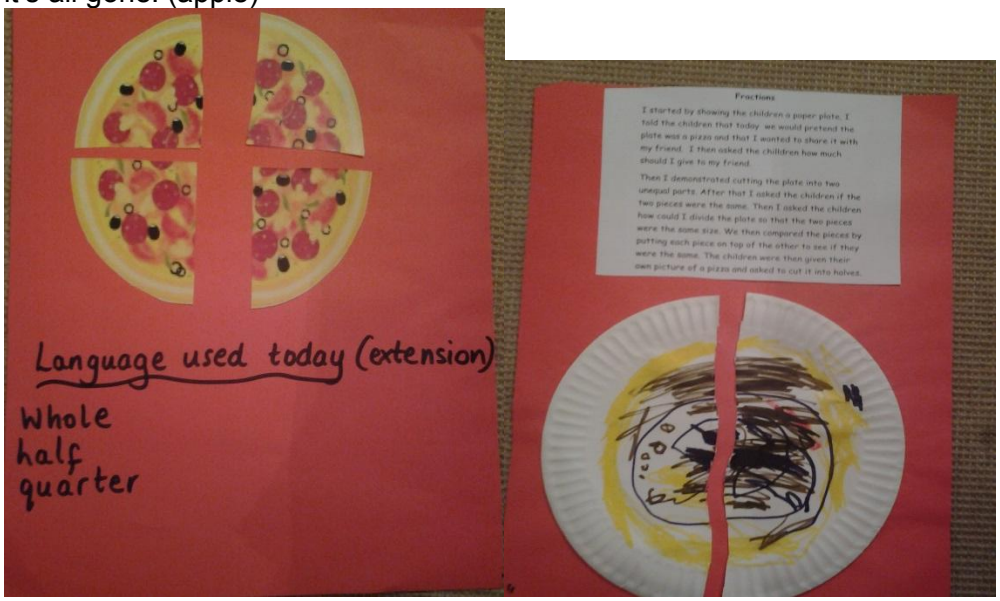
EYFS

Framework for EYFS 2012: They solve problems including doubling, halving and sharing.

Expanding this suggests that children would need to know that: Half is one of two equal parts if an object shape or quantity. Children need experiences of sharing into two equal parts. Extension onto raising ideas about 'what if' we had 3 children or 4 children.

Children's responses:

'Give some to each other'. 'It has to look the same.' ' You have a bite and I have a bite until it's all gone. (apple)'



How does this develop experiences in previous settings?

Emphasis on sameness and equality of shares assuming that they have done lots of informal sharing in pre-school. This means the children need to be able to compare numbers and quantities in a more sophisticated way that just saying bigger or smaller. How do you know the shares are equal? Intuitive ideas at this stage rather than formal proof using balance scales.

KS1: Y1/2

PoS:

Fractions Year 1

Pupils should be taught to:

- recognise, find and name a half as one of two equal parts of an object, shape or quantity
- recognise, find and name a quarter as one of four equal parts of an object, shape or

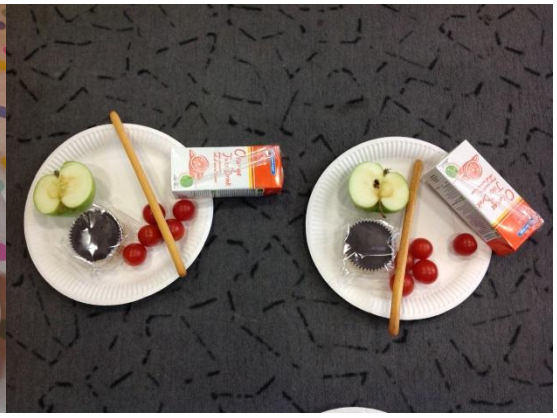
quantity.

Fractions Year 2

Pupils should be taught to:

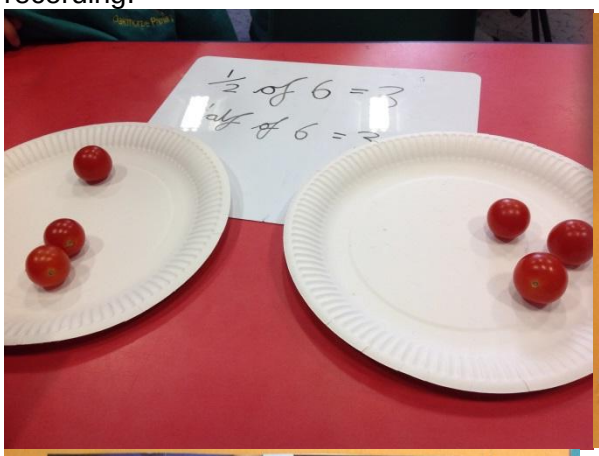
- recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity
- write simple fractions e.g. $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$

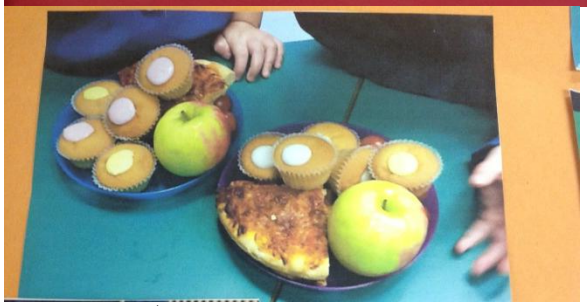
Children's responses



This is an example of a feast that was presented to the children and how they shared it between two children.

There is great importance of actually handling food and making the links between the articulation of the ideas and the 'stuff'- notion of $\frac{2}{4}$ being equivalent to $\frac{1}{2}$ before the symbolic recording.

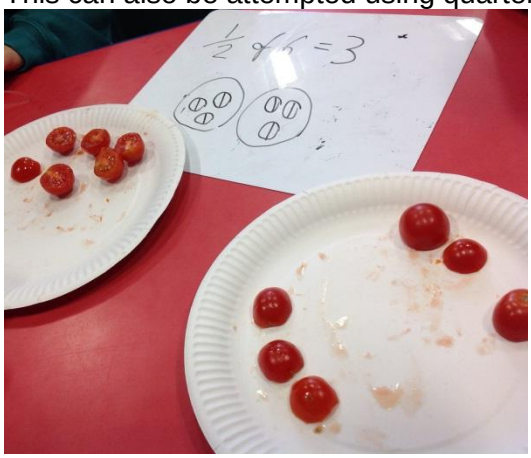




Talk about $\frac{1}{2}$ as one of two equal parts – move to symbolic representations is a sophisticated step- managing any symbolic recording is crucial. Move from showing one part out of two equal parts to the conventional recording.

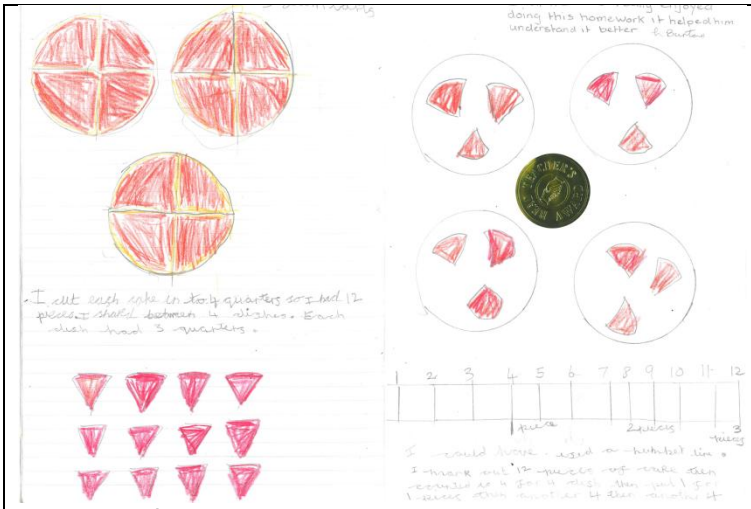
Begin counting in halves using concrete materials to make sense of the process. This will allow the children to understand and get a sense of fluency for how fractions are related to numbers.

Beginnings of a notion of quotient – rather than just sharing 6 tomatoes between 2, discuss how they might share the 6 tomatoes one at a time. This develops the idea with children that there are different ways to solve a sharing problem using fractions. It also helps the children to understand the 'whole' and how this relates to fractions.
This can also be attempted using quarters by quartering one tomato at a time.



Develop a sense of equality when relating to half and other fractions. How can we make sure that the food is shared fairly?

Present problems which ask children to use division where the answer will be a fraction (e.g. 3 jam tarts shared between 4 people). If this is worked through with the children practically they will be able to talk about how many quarters each person will get. This builds upon the notion of quotient explained above.



LKS2: Y 3/4

PoS

Fractions Year 3

Pupils should be taught to:

- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
- recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators
- recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators
- recognise and show, using diagrams, equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole (e.g. $\frac{5}{7}+\frac{1}{7}=\frac{6}{7}$)
- compare and order unit fractions, and fractions with the same denominators
- solve problems that involve all of the above.

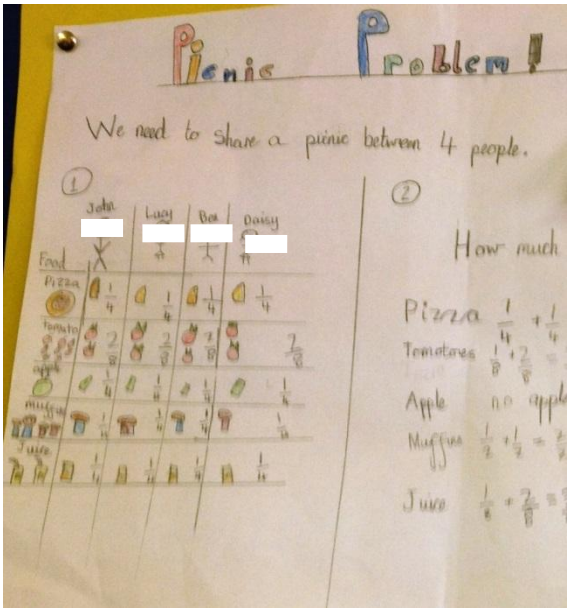
Fractions (including decimals) Year 4

Pupils should be taught to:

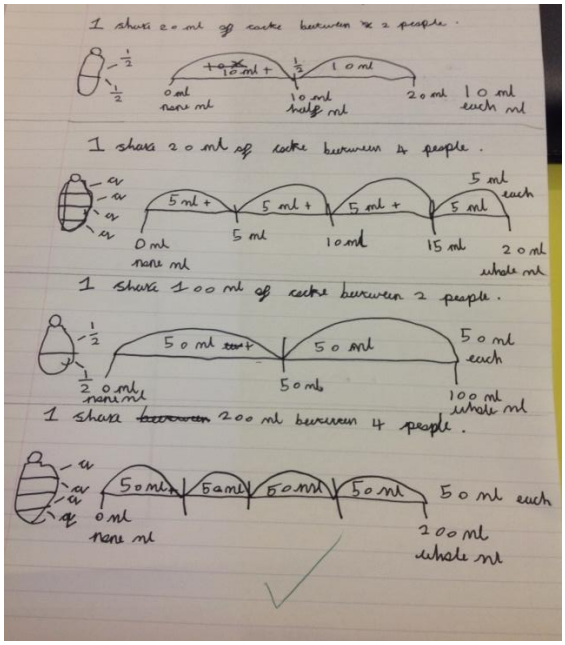
- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to to $\frac{1}{4}$; $\frac{1}{2}$; $\frac{3}{4}$
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places

Children's responses

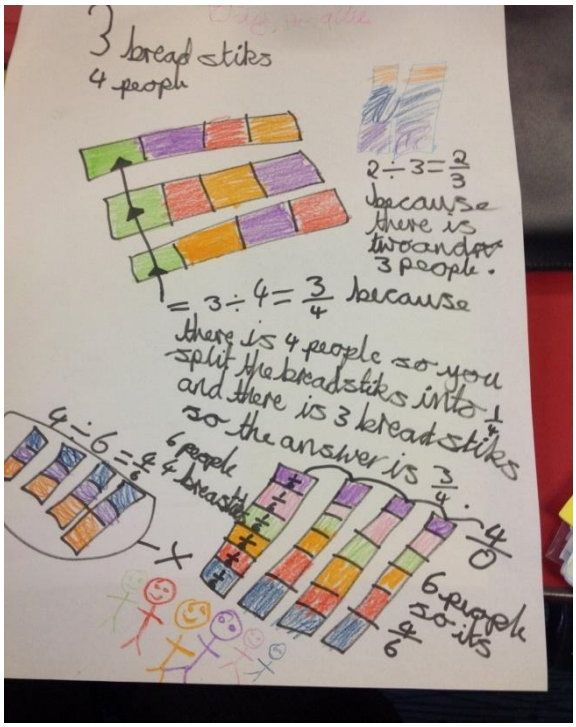
Begin to use tables to represent data – different ways of representing the same fraction: concrete, iconic, symbolic, words, images



Quotient begin to become an issue e.g. children saying things like – 'you can't divide one by two' but they have done this before. More sophisticated presentations of similar sharing problems e.g. have you ever shared 5 cakes between 8 people?
Fractions of quantities and links with measure – representations on number line



Dividing into larger numbers of shares. Moving away from halving and halving again to find $\frac{1}{3}$ so developing more sophisticated understanding of the denominator.



UKS2: Y 5/6

PoS

Fractions (including decimals and percentages) Year 5

Pupils should be taught to:

- compare and order fractions whose denominators are all multiples of the same number
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$]
- add and subtract fractions with the same denominator and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- read and write decimal numbers as fractions [for example, 0.71 = $\frac{71}{100}$]
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place
- read, write, order and compare numbers with up to three decimal places
- solve problems involving number up to three decimal places
- recognise the per cent symbol (%) and understand that per cent relates to 'number of

parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal

- solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25.

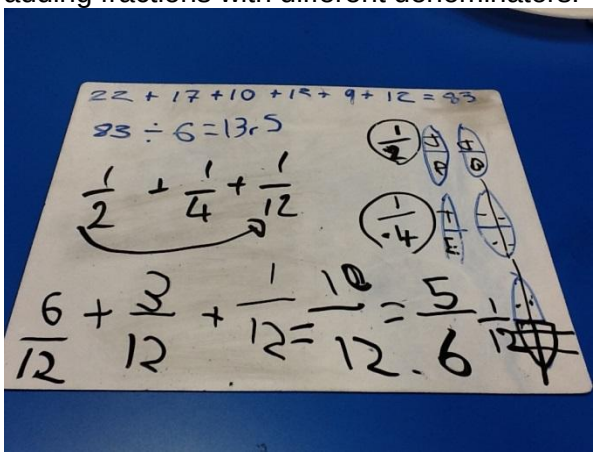
Fractions (including decimals and percentages) Year 6

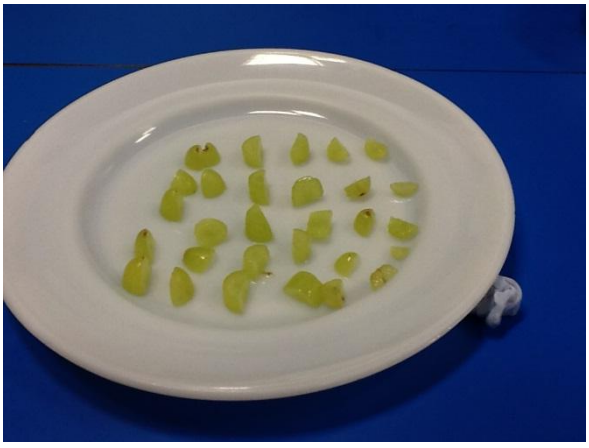
Pupils should be taught to:

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]
- divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$]
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$]
- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

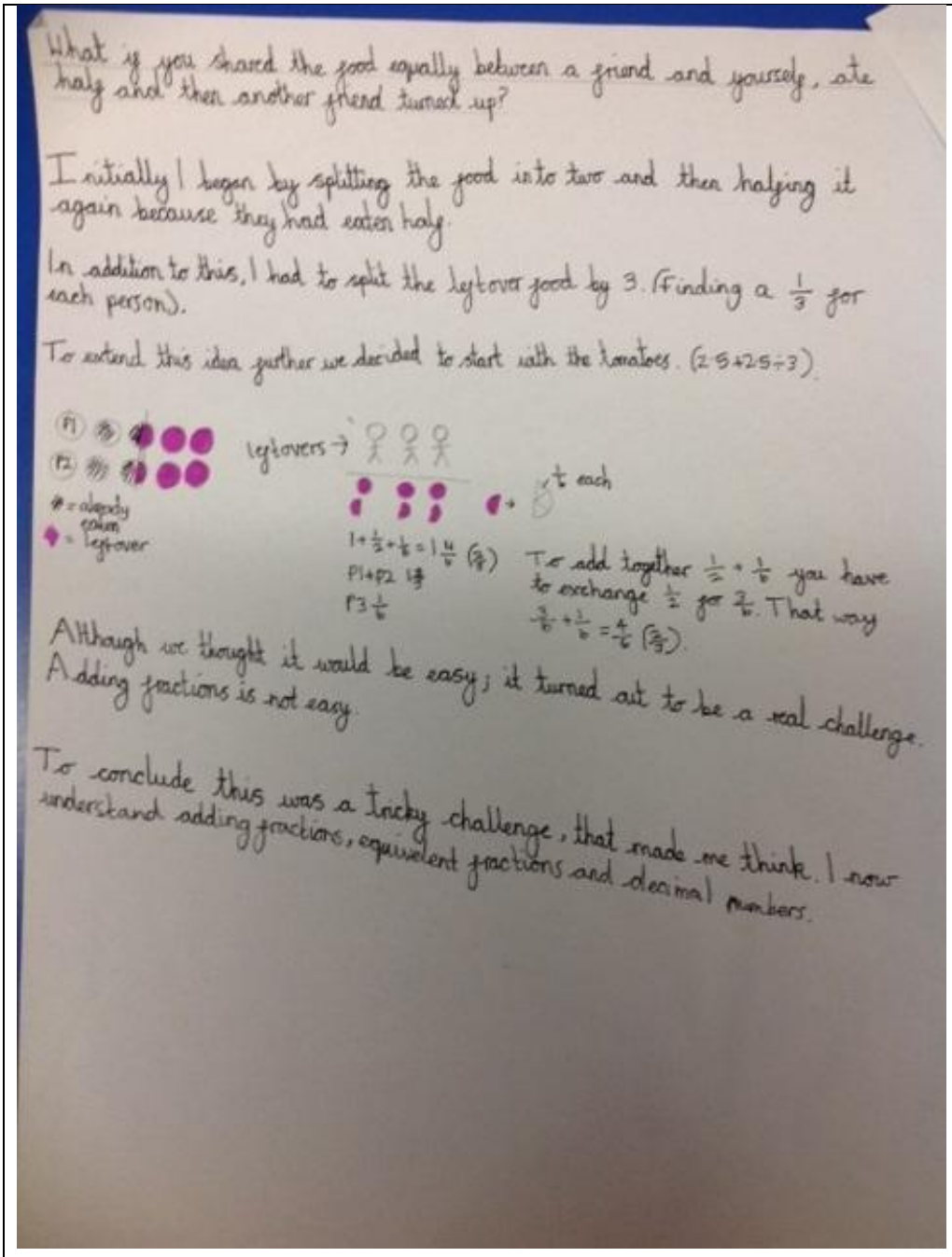
Children's responses

Punnet of grapes – using the real grapes for children to solve problem practically. 83 grapes divided between 6 people– finding fractions of large quantities and expressing the result in mixed numbers – some split each grape into sixths and another halves, then quartered and then divided by twelve – exploring the answers and their equivalence led to adding fractions with different denominators.

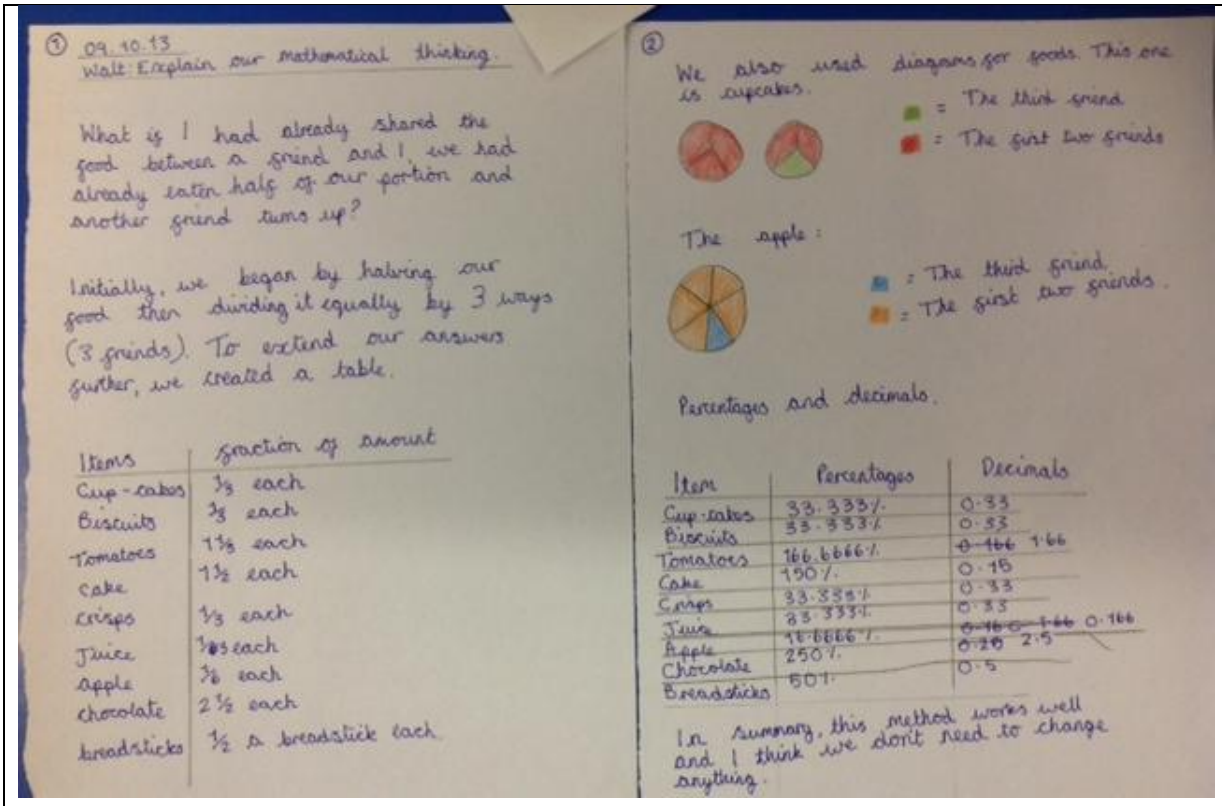




Quotients much more prevalent – sharing the food between two and had eaten half a portion when another friend turns up.



Percentages and decimal equivalents



① 09.10.13
Walt: Explain our mathematical thinking.

What is 1 had already shared the food between a friend and I, we had already eaten half of our portion and another friend turns up?

Initially, we began by halving our food then dividing it equally by 3 ways (3 friends). To extend our answers further, we created a table.

Items	fraction of amount
Cup-cakes	⅓ each
Biscuits	⅓ each
Tomatoes	1⅓ each
Cake	1½ each
Crisps	⅓ each
Juice	⅓ each
Apple	⅓ each
Chocolate	2½ each
Breadsticks	½ a breadstick each

②

We also used diagrams for goods. This one is cupcakes.

= The third friend
= The first two friends

The apple:

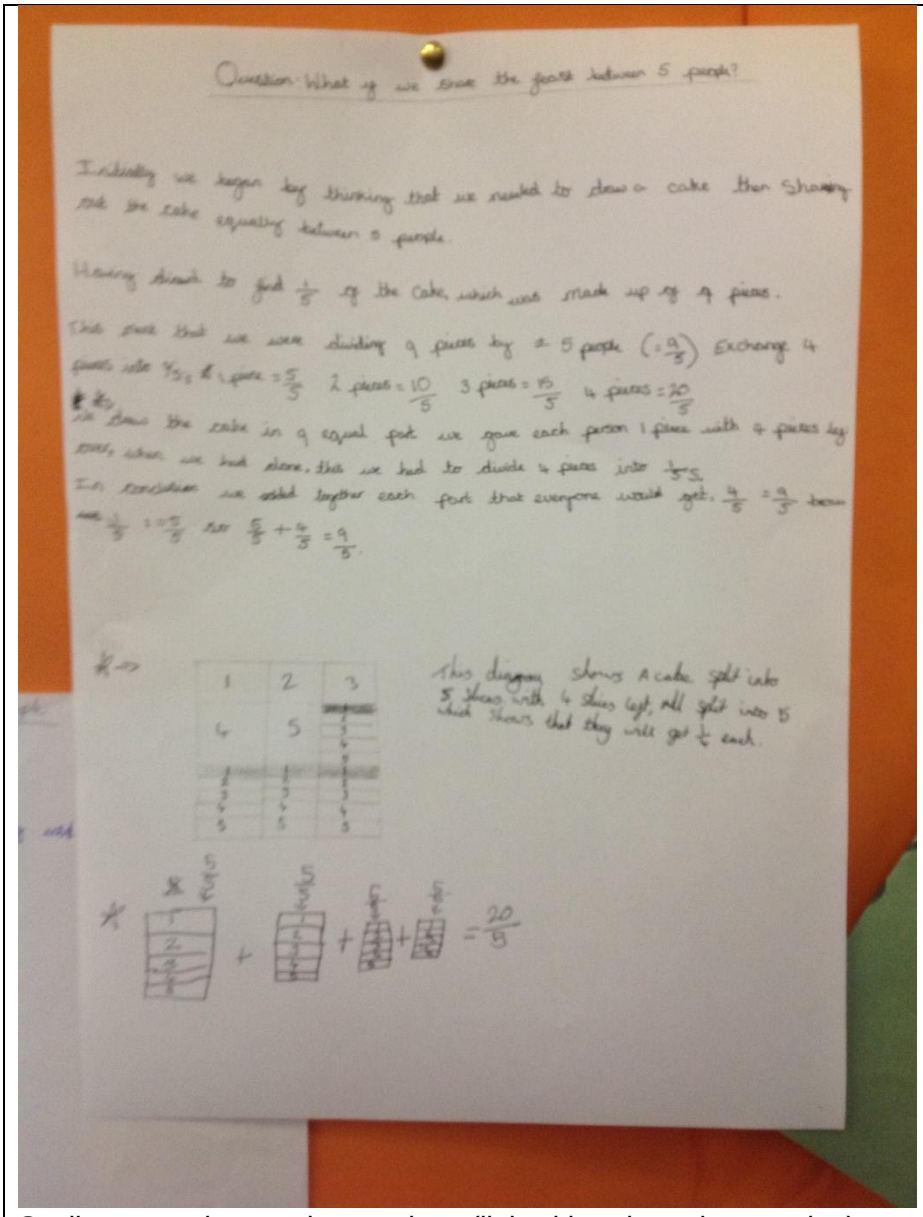
= The third friend.
= The first two friends.

Percentages and decimals.

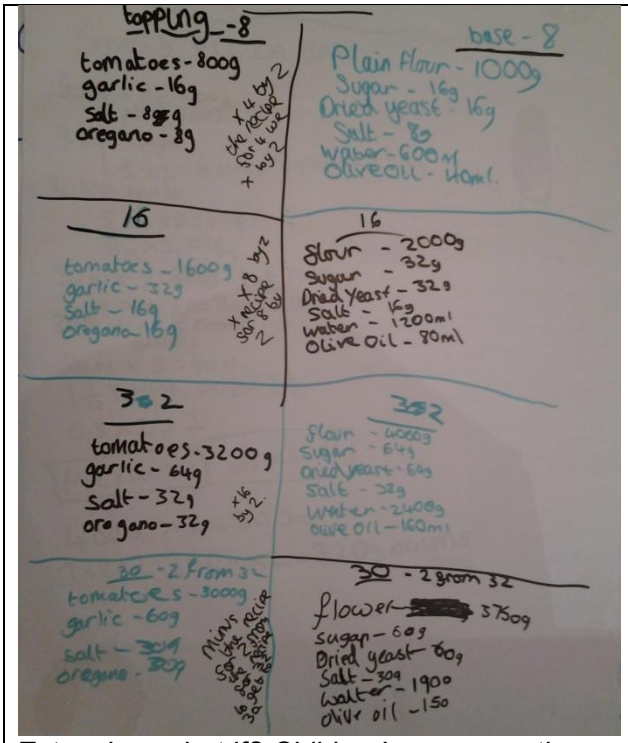
Item	Percentages	Decimals
Cup-cakes	33.333%	0.33
Biscuits	33.333%	0.33
Tomatoes	166.6666%	~~0.166~~ 1.66
Cake	150%	0.15
Crisps	33.333%	0.33
Juice	83.333%	0.33
Apple	16.6666%	~~0.16~~ ~~1.66~~ 0.166
Chocolate	250%	~~0.25~~ 2.5
Breadsticks	50%	0.5

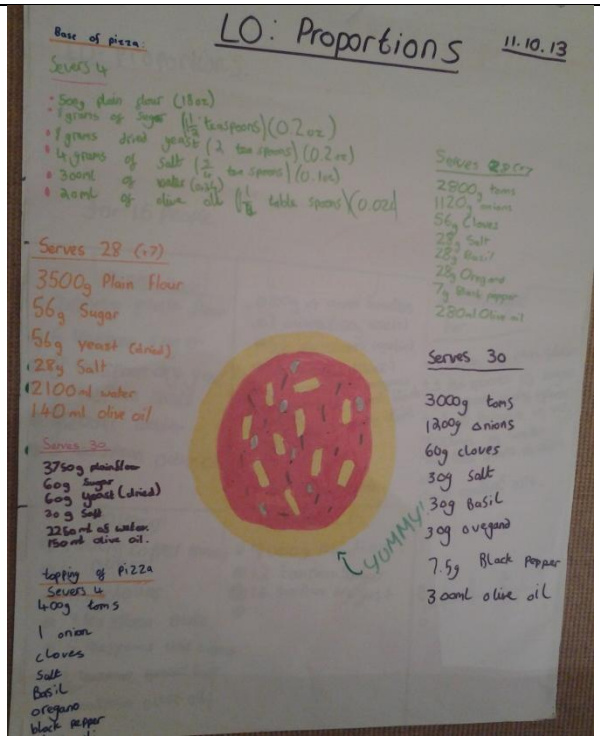
In summary, this method works well and I think we don't need to change anything.

Adding fractions; Example of taking the cake which had been divided in to 9 equal shares and then giving each of 5 people 1/9 and then four equal shares of the remaining $\frac{4}{9}$ i.e. $\frac{4}{45}$ each – this is equivalent to $\frac{1}{5}$ as $\frac{1}{9} + \frac{4}{45} = \frac{1}{5}$

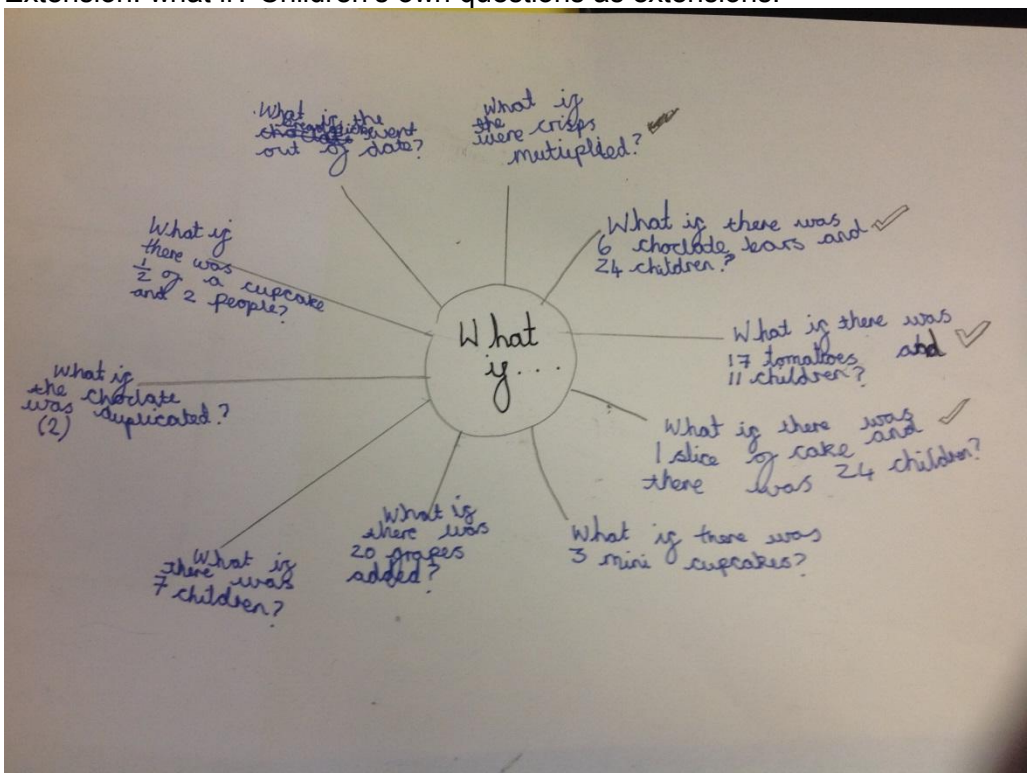


Scaling up recipes – pizza recipes (link with ratio and proportion)





Extension: what if? Children's own questions as extensions.



KS3

Different aged people needing different shares e.g. adult, teenager, child

Key extension here is moving into proportional reasoning at a deeper level.

Scaling up recipes – posing questions on recipe examples such as what is the relationship between the recipe for 8 and for 30 people?

Appendix 2

Case studies of focus pupils

Pupil A Case Study

Pre test score: 8/39	**Post test score:19/39**

How has this child made progress?
(attitude towards fractions, understanding of fractions, application of fractions, misconceptions overcome)
Pupil A approaches fractions enthusiastically and is happy to attempt problems involving fractions involving whereas previously they would have left them unattempted. Initially they recorded the fraction incorrectly (denominator mixed up with the numerator), this has now been rectified. They are able to show a full understanding of part whole fractions and is beginning to simplify fractions to their lowest form. They are able to find fractions of numbers and confidently draw their own visual representations to support understanding and solving of problems. They are able to relate some fractions to decimals and percentages.

What has been the biggest influence on this child's progress?
(tasks, resources, representations, intervention)
The rich task widely influenced this child's progress as they were able to discover and learn fraction skills for themselves, allowing the time to explore fractions. Using visual representations in different formats and showing how fractions can be represented in different ways has given them a new way to represent these and a deeper understanding of what this means / looks like.
Having lots of smaller group sessions with a teacher has helped identify misconceptions for this child and target intervention as well as spending more time on fractions than you would usually.

What improvements have they shown in use of visual representations?
(pick out examples from the test paper)
Pupil A is able to identify where the same fraction can be visually represented in different ways and is beginning to make links with this to equivalent fractions. (question 14)
They use visual representations to help further understand problems – e.g. question 17 / 19
Pupil A is able to draw visual representations to help support their learning of mixed numbers – e.g. question 30 and 22 and now shows a firm understanding of how to represent this visually.

They use visual representations to help solve quotient questions (question 25).

What improvements have you seen in their fluent use of fractions?
(What evidence have you got to support this?)
Pupil A is able to make further links between different areas of fractions – e.g. using part whole visual representations to solve equivalent fraction problems.
Pupil A uses visual representations to solve linear problems – e.g. 8 / 4 – they drew this visually and this helped them to understand where it would be placed on the number line.

Pupil B Case Study

Pre test score: 18/39	**Post test score: 27/39**

How has this child made progress? (attitude towards fractions, understanding of fractions, application of fractions, misconceptions overcome) Pupil B has shown increased confidence when handling fractions. Pupil B was quieter than some during the 'Fair Feast' task but did contribute.
What has been the biggest influence on this child's progress? (tasks, resources, representations, intervention) Fair Feast was a non-threatening way in. Involvement in a group and not just one child. Visual representations of one fifth of a set helped to find two fifths of a set. Intervention opportunity meant that we stopped when unsure of non-unit fractions and when back to simple fractions of a set.
What improvements have they shown in use of visual representations? (pick out examples from the test paper) Pupil B has not particularly used own visual representations in either test but does use them in class more. In the second test did attempt using own visual representation to solve $\frac{1}{5} + \frac{2}{5}$ but still came up with $\frac{3}{10}$ as the answer. But this is progress from the pre-test.
What improvements have you seen in their fluent use of fractions? (What evidence have you got to support this?) Part-whole – visual and quantity is well developed and more fluent – evidence from work in books. Need to further develop linear representations. Can attempt with simple fractions and mixed numbers but not improper fractions Improved with the use of quotient but not fluent.

Pupil C Case Study

Pre test score: 7/39	**Post test score:14/39**
How has this child made progress? (attitude towards fractions, understanding of fractions, application of fractions, misconceptions overcome) Pupil C has got a more positive attitude towards fractions and is more confident. They have developed a really good understanding of quotient fractions. Their understanding (according to the questionnaire) has moved from 4 to 9 (out of 10).	
What has been the biggest influence on this child's progress? (tasks, resources, representations, intervention) The use physical things (cups, food etc) It has helped that a lot of the activities were based on real life context to apply learning to. Also during focused group time they were able to discuss ideas, calculations and worries one-to-one. Having one-to-one meant that there was more time to focus on any issues and talk through them to find the correct answers.	

What improvements have they shown in use of visual representations?
(pick out examples from the test paper)
In the first test they drew circles and it seemed they didn't know why (e.g. How many thirds in a whole? – drew a circle and drew three lines to split the circle into sixths).
In the second test they logically used drawings to help (e.g. 4 cakes shared between 5 friends – drew 4 cakes, shared them into fifths and then draw the people to share out the fifths.)

What improvements have you seen in their fluent use of fractions?
(What evidence have you got to support this?)
They have a much better understanding of quotient and can explain where their ideas are coming from (this is due to the rich task and putting the context of fractions into a real life scenario). At the beginning of the project they had no idea how to calculate with fractions and they seemed to frighten them. After the fractions day they will now have a go at any fractions activity.

Pupil D Case Study

Pre test score: 12/39	**Post test score:17/39**

How has this child made progress?
(attitude towards fractions, understanding of fractions, application of fractions, misconceptions overcome)
Improved confidence towards fractions – able to see link between equivalent fractions - $\frac{2}{4}$ and $\frac{1}{2}$ are the same because 2 ÷ 1 = 2 and 4 ÷ 2 = 2.

What has been the biggest influence on this child's progress?
(tasks, resources, representations, intervention)
Representations - to begin with only confident with part-whole (e.g. )
Now able to use part-whole quantity (e.g. 

What improvements have they shown in use of visual representations?
(pick out examples from the test paper)
Opportunity for reasoning about what they see.
 in the pre-test this would have been $\frac{1}{3}$ but in the second test it was answered as $\frac{3}{4}$.
Now I know its $\frac{3}{4}$ because 3 are shaded out of a total of 4.

What improvements have you seen in their fluent use of fractions?
(What evidence have you got to support this?)
Able relate to multiplication and division. I know $\frac{3}{6}$ is the same as $\frac{1}{2}$ because 3 is half of 6.

Pupil E Case Study

Pre test score: 7/39	Post test score:20/39

How has this child made progress?
(attitude towards fractions, understanding of fractions, application of fractions, misconceptions overcome)
More willing to attempt fractions questions.
Much more able to find a fraction of an amount
Part-whole used in different ways
Quotient - thinking about $\frac{3}{4}$ as 3 pizzas shared between 4 people and so how much of the total amount they would have. Most of these questions are now answered correctly.

What has been the biggest influence on this child's progress?
(tasks, resources, representations, intervention)
Intervention group allow explanation of thinking and sharing of ideas.
Discussion in groups of how others have represented questions helps them to explain what resources and drawings they have used.
Small group work allowed more focus on the task.

What improvements have they shown in use of visual representations?
(pick out examples from the test paper)
Question 25 (quotient) – drew the chocolate bars and divided one bar in twelves and worked out all would get $\frac{8}{12}$.
Question 21 (quotient) – draw pictures and solved similar to above
Chocolate drawn as rectangles. Others drawn in different ways.
Images were also drawn for other questions to show thinking of what the fraction looks like.

What improvements have you seen in their fluent use of fractions?
(What evidence have you got to support this?)
More evidence of using multiplication and division. Initial group ideas was to cut each cake into sevenths to work out how many sevenths each would get. This has moved away from finding half, quarter, eighths as a default way to find a fraction of a shape.

Photography/Video at your school

Contact
Contact from the NCETM will be either through a member of our team or one of your teachers who takes part in our events. If required, we are happy to explain to the head teacher what we do when we carry out a photograph/video shoot in school, and about data security. Once we have discussed with the teacher what class will be involved in the shoot, we will ask for each student to have a consent form completed and returned to the school; we are required to hold a copy of the signed form.

Consent
Each pupil will be issued with a consent form to take home to be completed by their parent/guardian, indicating that they are happy for them to take part in the shoot. The teacher will also be required to complete the consent form *(attached to this document)*: this ensures that the school and the NCETM are complying with the Data Protection Act.

Usage
We use images in promoting and advertising our aims in publications and on the NCETM web portal. Video is used on our web portal, on our YouTube channel, and any other NCETM-approved medium.

Safety
We understand that the safety of the children is paramount and we take every step to ensure we do not put children in danger:

Imagery
- each image used of a child has no identification of where the child attends school or the location of the school;
- if the images are to be used with school identification, the child's image is blurred or cropped so the child cannot be identified;
- if anyone is not happy to take part in the shoot we will not include that student in any images shot. They will still take part in the class as normal;
- to ensure no-one is included in the shoot that doesn't have consent we will ask the teacher to identify the students to the NCETM photographer and if possible work in a separate group.

Video
- although every effort is made to minimise the risk of a child being identified in connection with their school, it is impossible to totally eliminate this risk, due to the nature of video. Therefore the child may be identifiable;
- all care will be taken while videoing to avoid identifying the school from logos on clothing or school signs where possible. In such cases where this is impossible, every effort will be made to avoid close-up shots of the face.

The School
We are happy to provide two large printed images and/or these digital files from the photographic shoot to the school for their own usage.

Contact
If you have any questions please contact us at info@ncetm.org.uk or on 0114 219 1007.

Photography/Video Consent Form

Name of person being photographed:	School/College/Institution Name:
Class: Teacher:	School Address
	Tel no:

Agreement is given for the above to take part in a photography/video shoot for The National Centre for Excellence in the Teaching of Mathematics (NCETM).

I consent to the full use of the material by the NCETM in any form and in any medium, which reasonably promotes or advertises the aims of the NCETM without time limit, without liability or acknowledgement to me as required under the Copyright, Designs and Patents Act 1988 (the 'Act') or otherwise. I hereby confirm that all moral rights under the Act or otherwise have been waived.

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Print Name……
(signed by Parent or Guardian, if under 18. In so signing, the Parent or Guardian accepts that he/she and the minor are bound by all the provisions of this document.)

Date…….