Triangles in Circles

nrich

9-Dots

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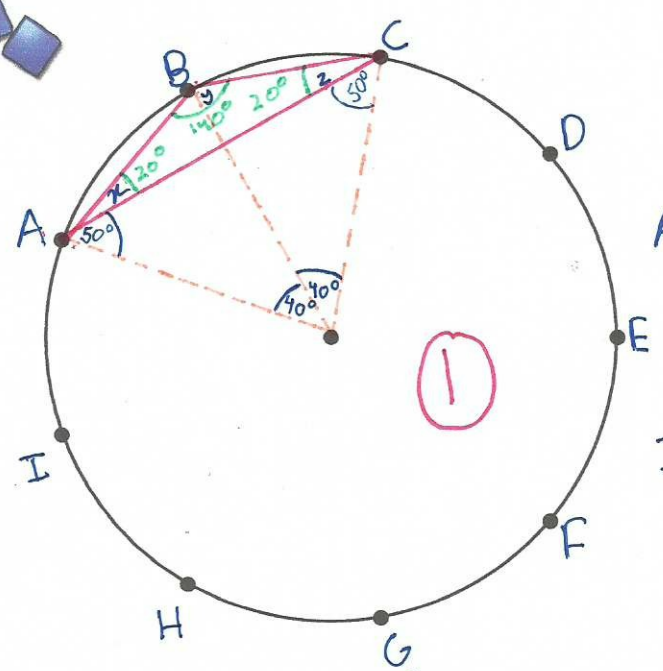
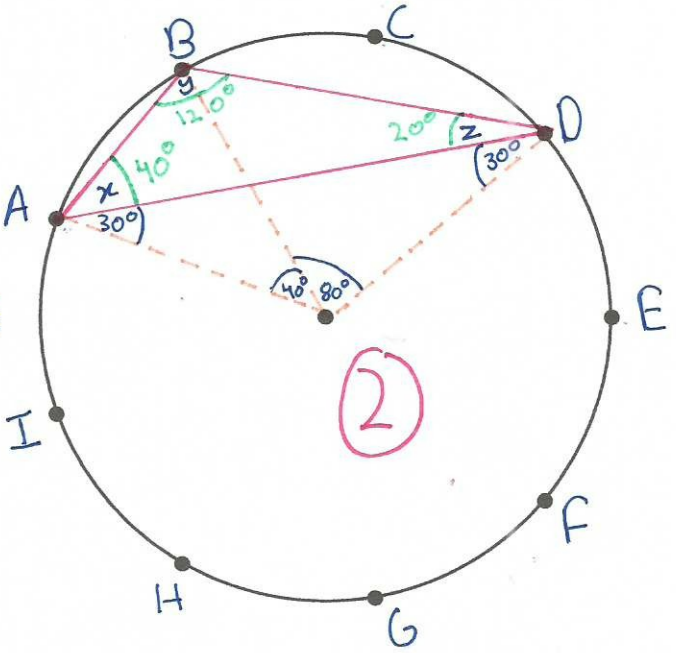
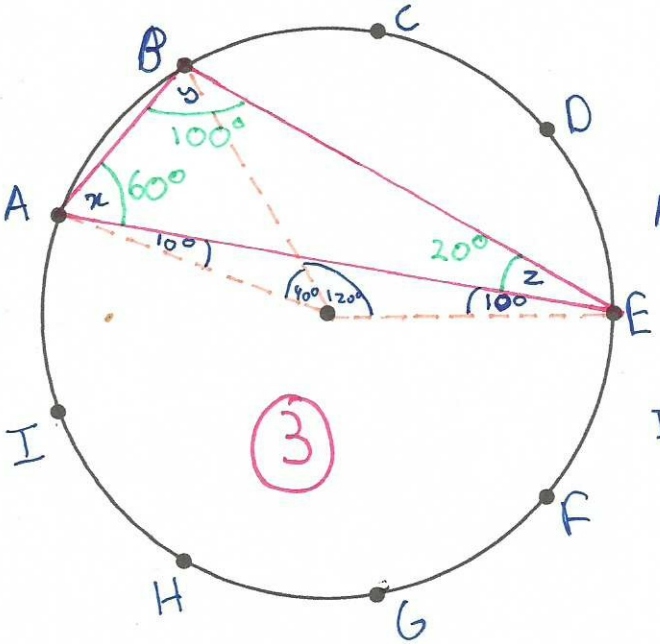
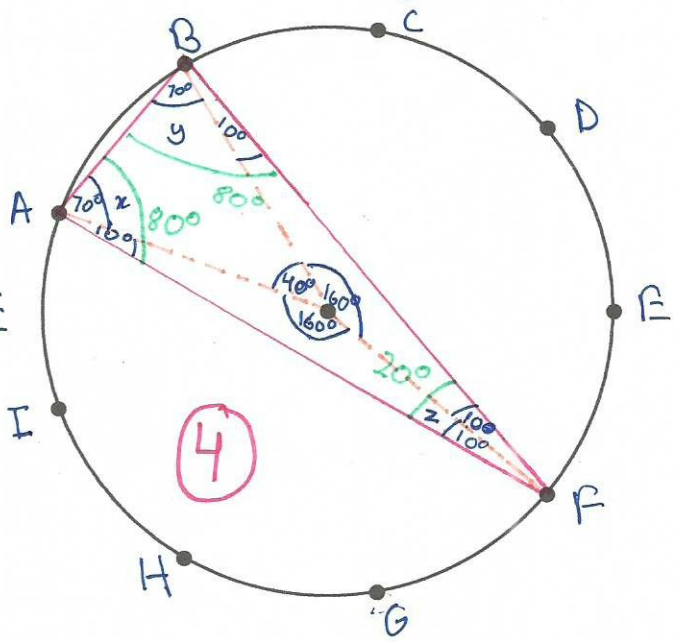
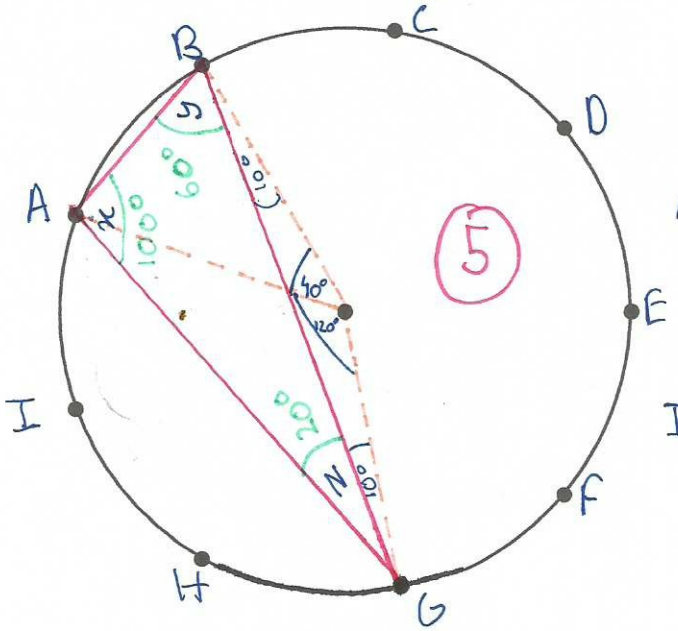
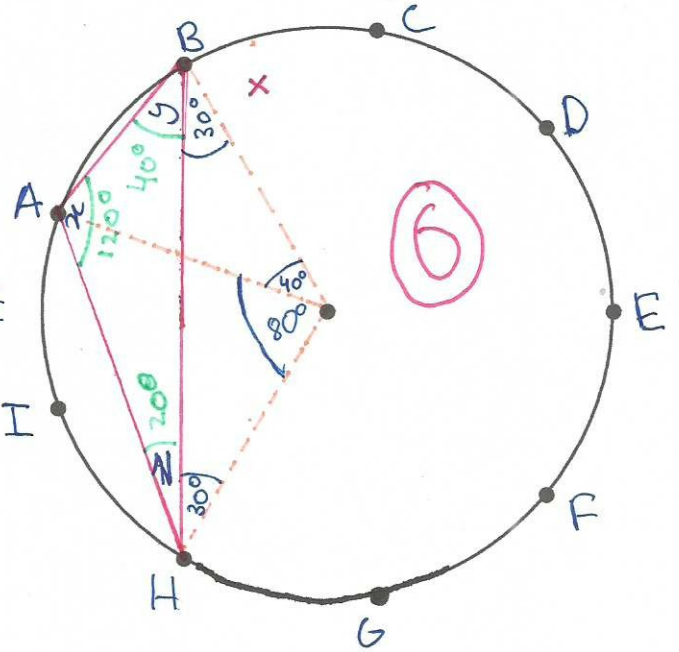


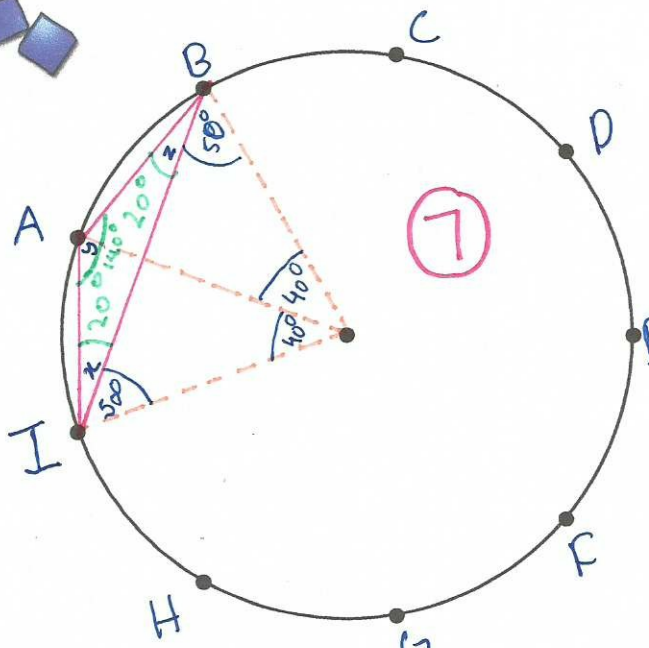


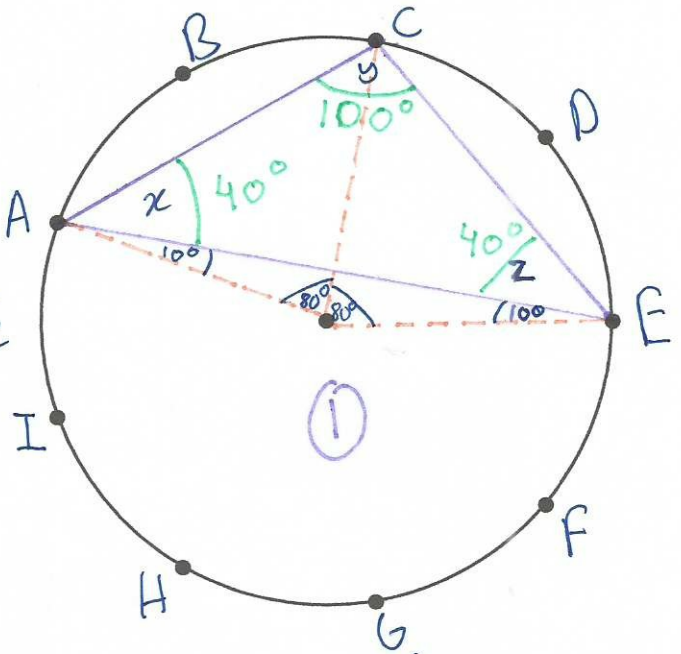


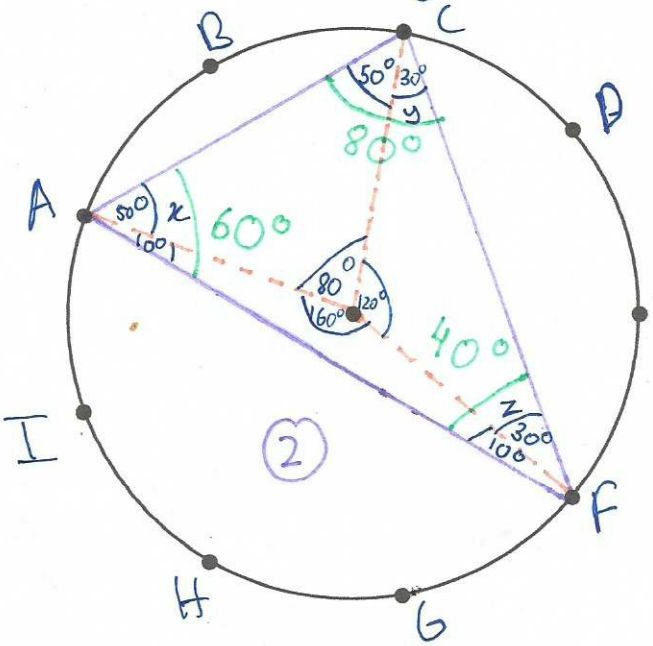


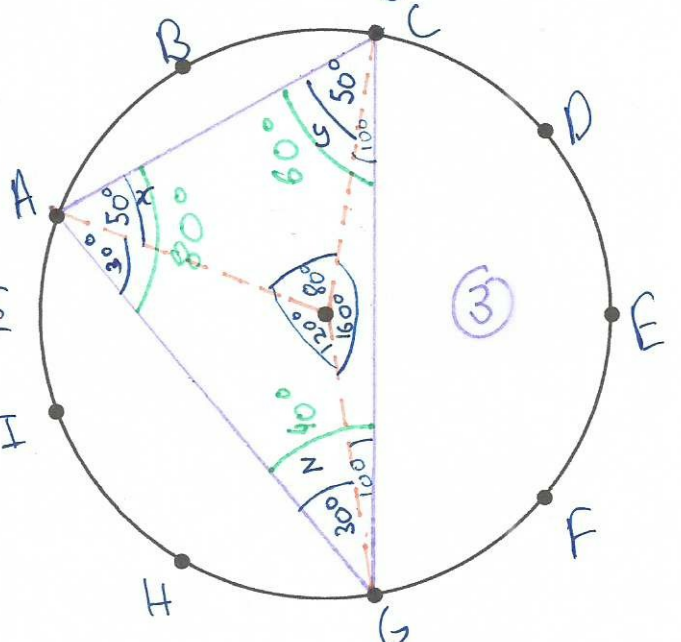


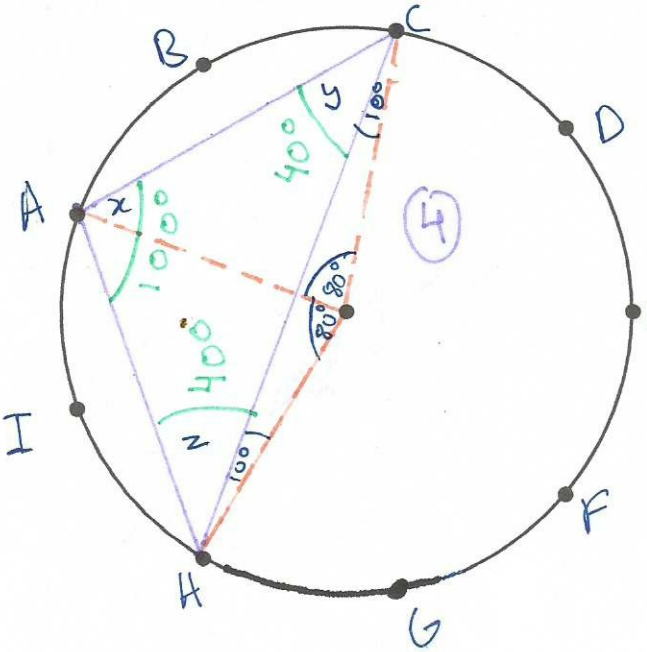


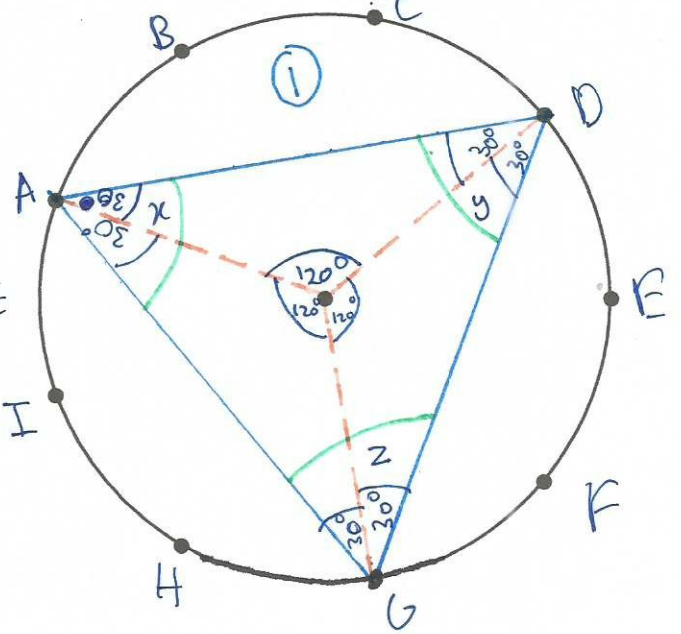

A
B
C
D
E
F
G
H
I
7


A
B
C
D
E
F
G
H
I
x
y
z
40°
100°
1


A
B
C
D
E
F
G
H
I
2


A
B
C
D
E
F
G
H
I
3


A
B
C
D
E
F
G
H
I
4


A
B
C
D
E
F
G
H
I
1

To compare our results with the different triangles, we put them in a table to make it clear. We start with a base of AB.

	1	2	3	4	5	6	7	
∠	△ABC	△ABD	△ABE	△ABF	△ABG	△ABH	△ABI	×
x	20°	40°	60°	80°	100°	120°	20°	×
y	140°	120°	100°	80°	60°	40°	140°	×
z	20°	20°	20°	20°	20°	20°	20°	×

We now understand that some of the triangles above have been repeated so we can now separate them to make unique triangles.

Unique Triangles	Repeats
1. ABC	1. ABC / ABI
2. ABD	2. ABD / ABH
3. ABE	3. ABE / ABG
4. ABF	

Then we move on to a base of AC.

	1	2	3	4
∠	△ACE	△ACF	△ACG	△ACH
x	40°	60°	80°	100°
y	100°	80°	60°	40°
z	40°	40°	40°	40°

Just like for the other set, we find repeats. Here is the list again.

Unique Triangles	Repeats
1. ACE	1. ACE / ACG
2. ACF	2. ACF / ACH

Lastly, we have an equilateral triangle which is (obviously unique).
So . . .

Unique Triangles	Repeats
1. A D G ①	None

Conclusion

There are 7 unique triangles, all with different angles and no repeats. They are...

1. A B C
2. A B D
3. A B E
4. A B F
5. A C E
6. A C F
7. A D G