Joshua Luke Tutin

2 player version of the factors multiples game and solving first 50+ single player factors multiples games

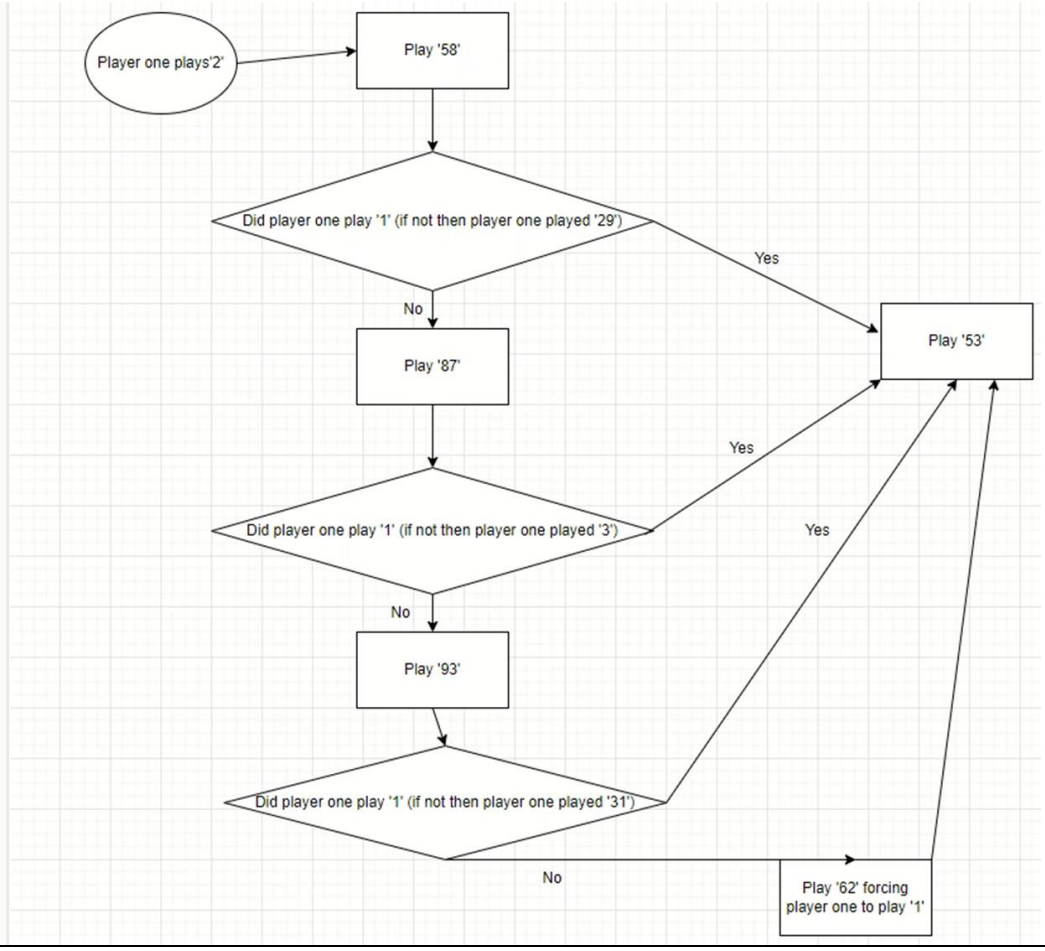
Numbers to avoid for winning the factors multiples game (numbers 1 to 100):

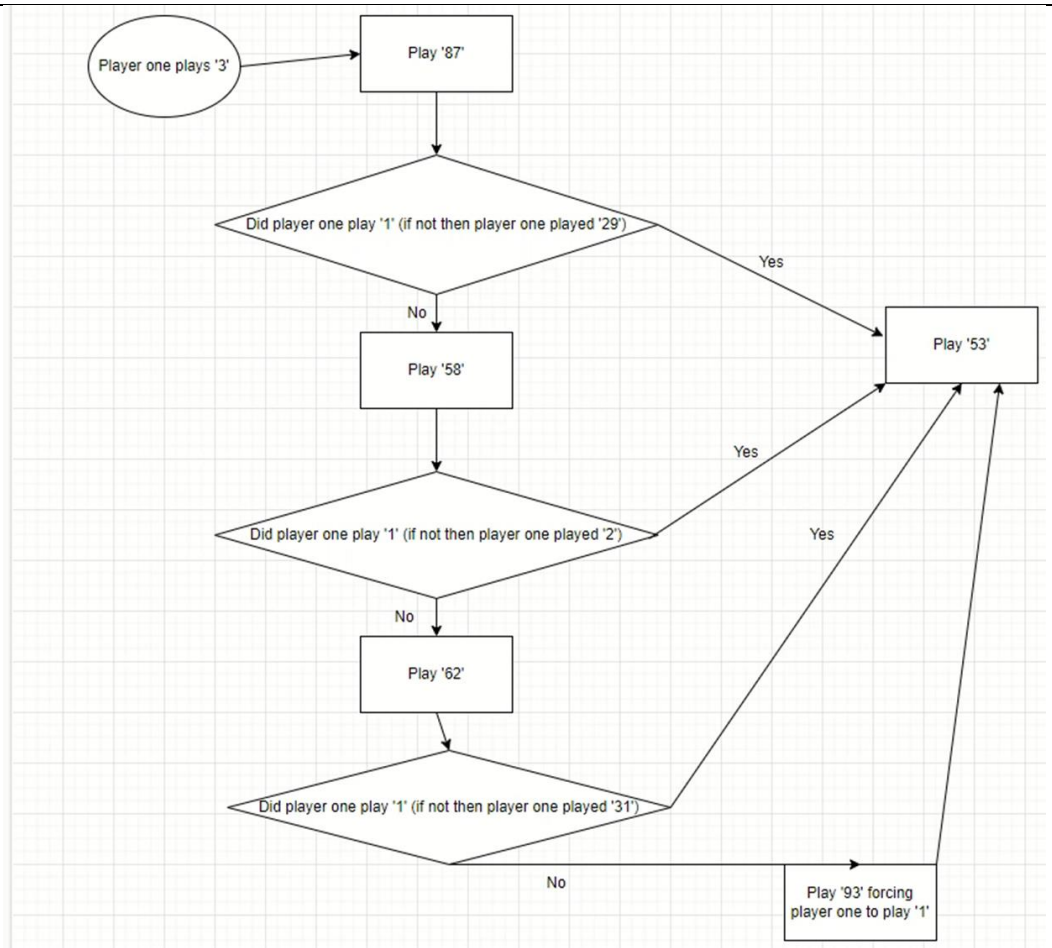
Prime numbers less than 50 (since multiplying prime numbers under 50 to get numbers over 50 makes it easy to counteract them).

The factors multiples game (numbers 1 to 100) is a game about trying to force your opponent to play either 1 or a prime number under 50.

Possible future technique for fully solving the factors multiples game (numbers 1 to 100):

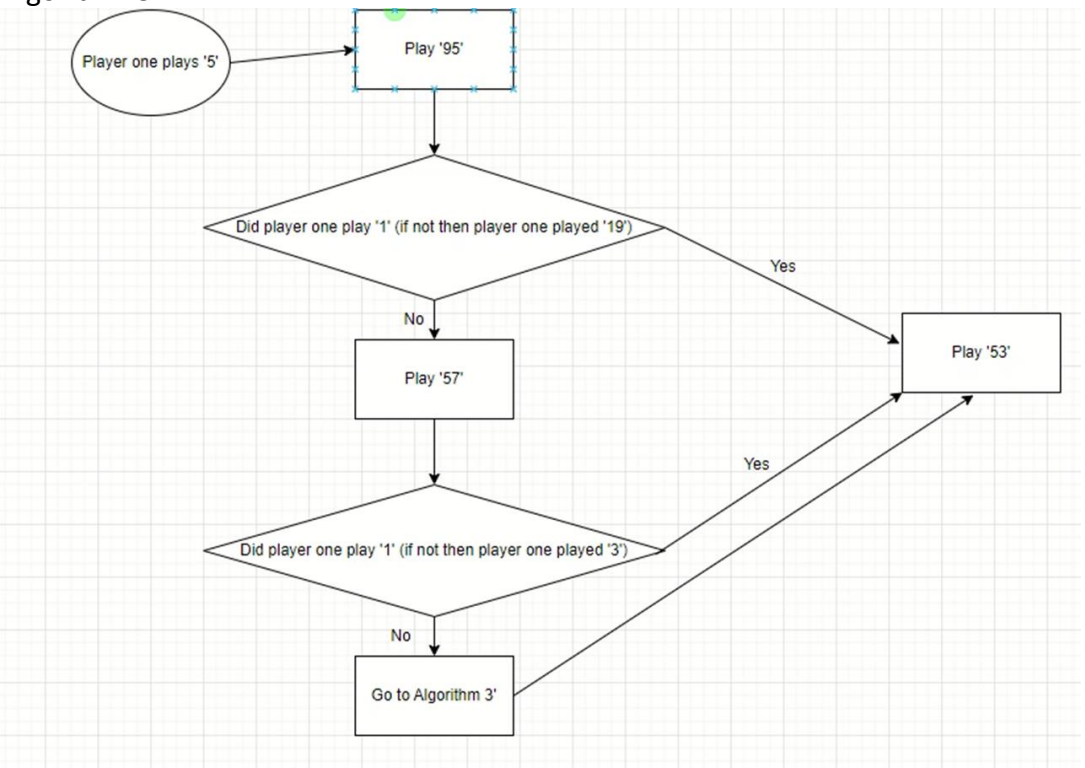
Retrograde analysis (https://en.wikipedia.org/wiki/Retrograde_analysis) to work backwards from prime numbers below 50 to find all winning/losing numbers.

Prime for player one	Winning strategy for player two
2	Algorithm 2: 
3	Algorithm 3:



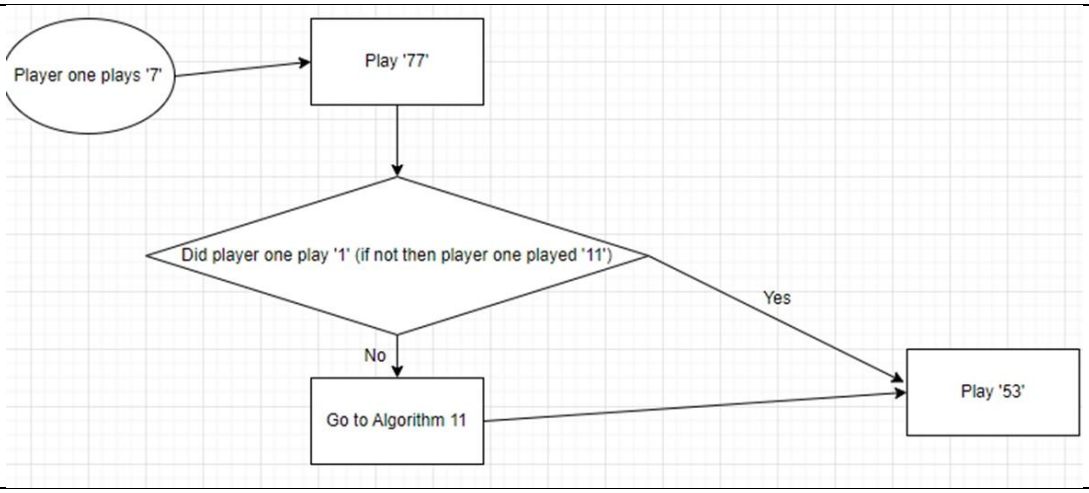
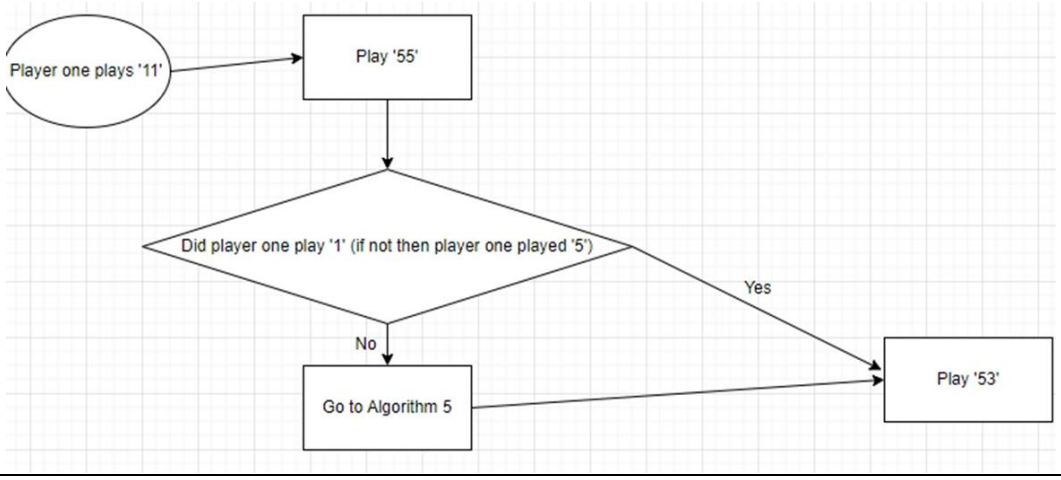
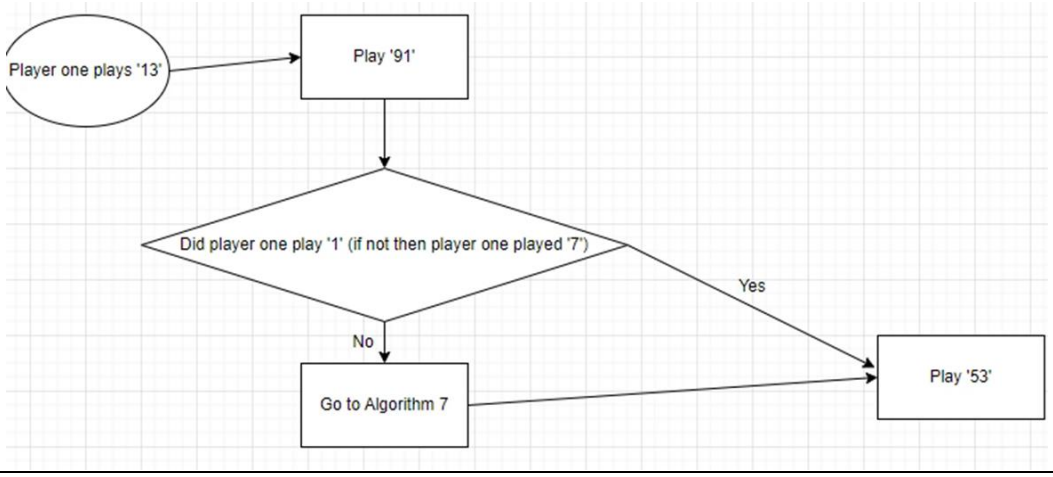
5

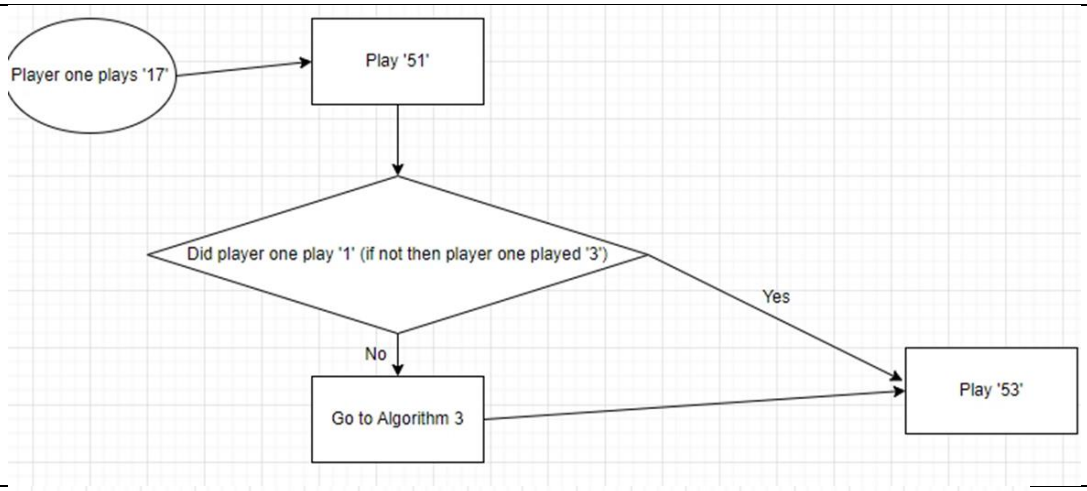
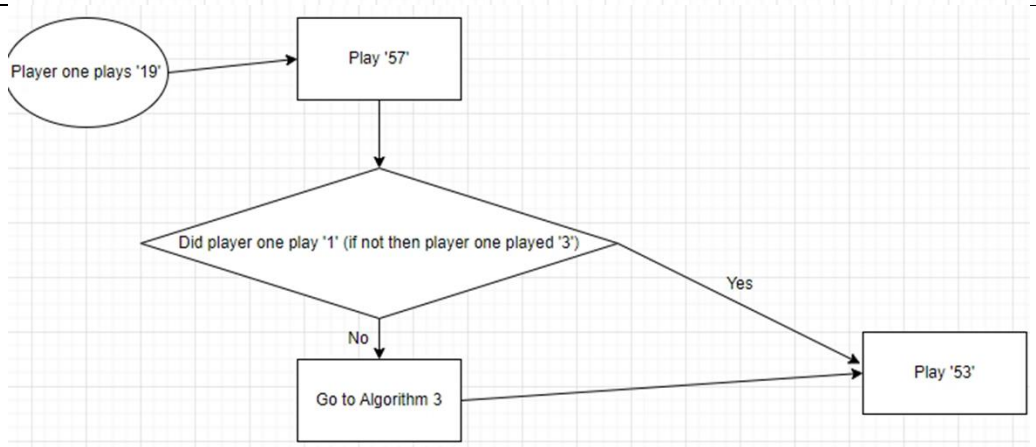
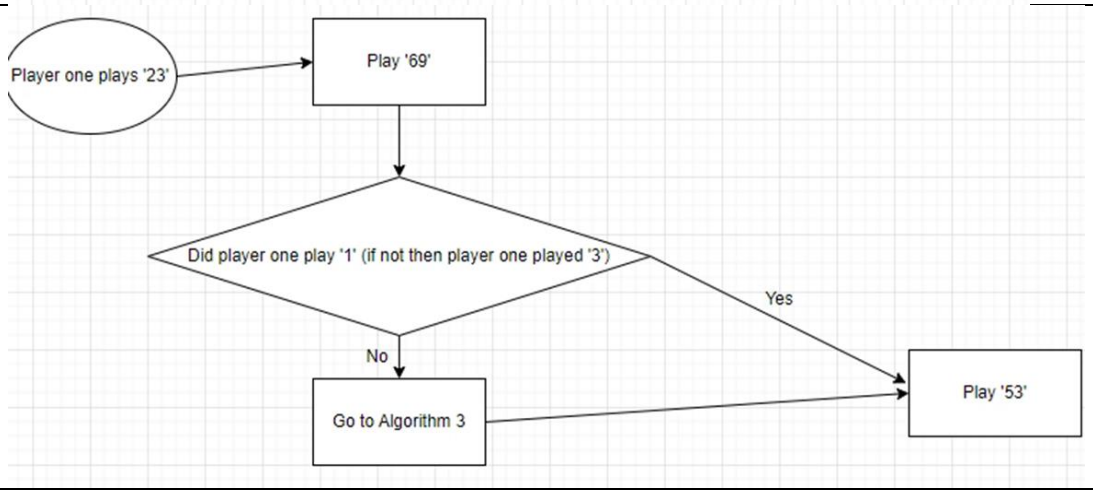
Algorithm 5:



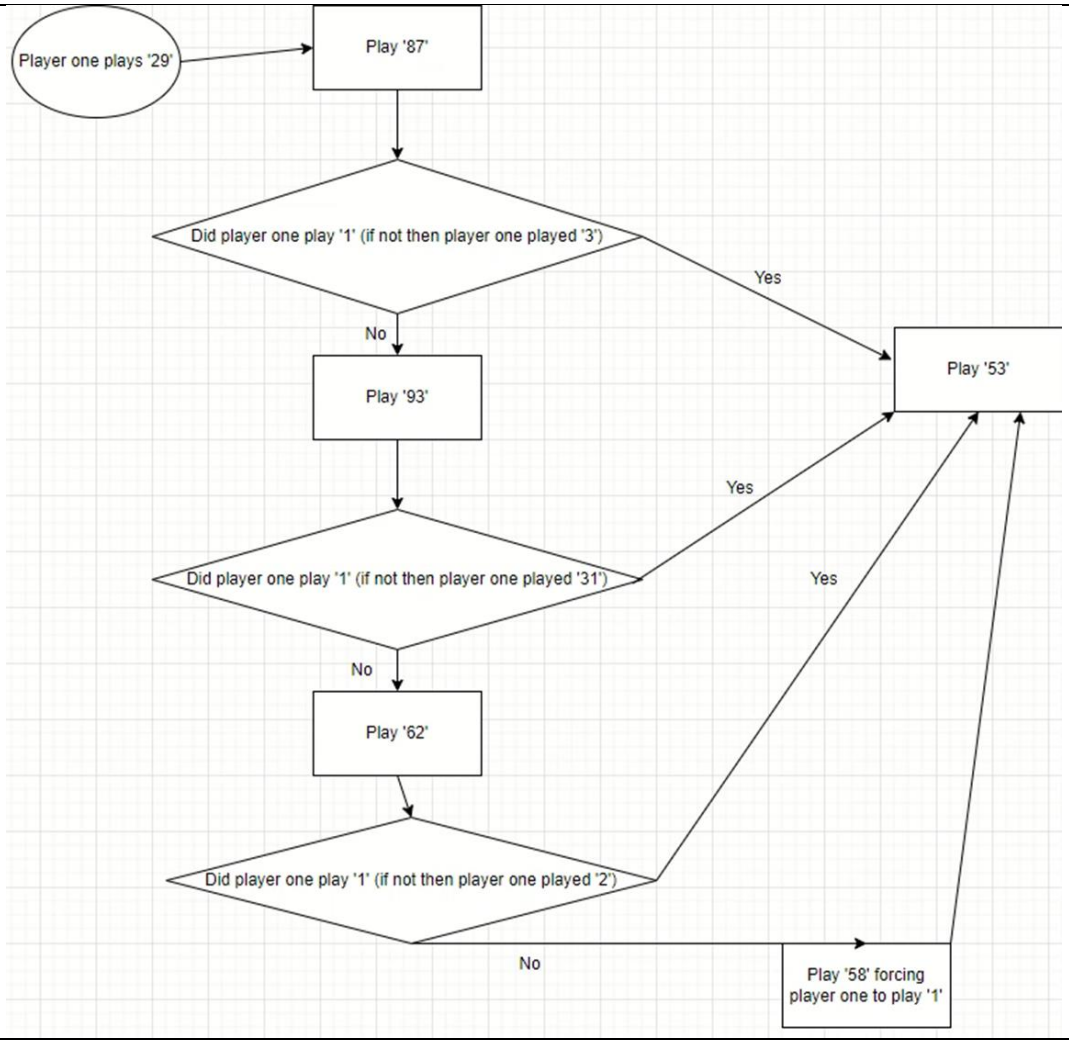
7

Algorithm 7:

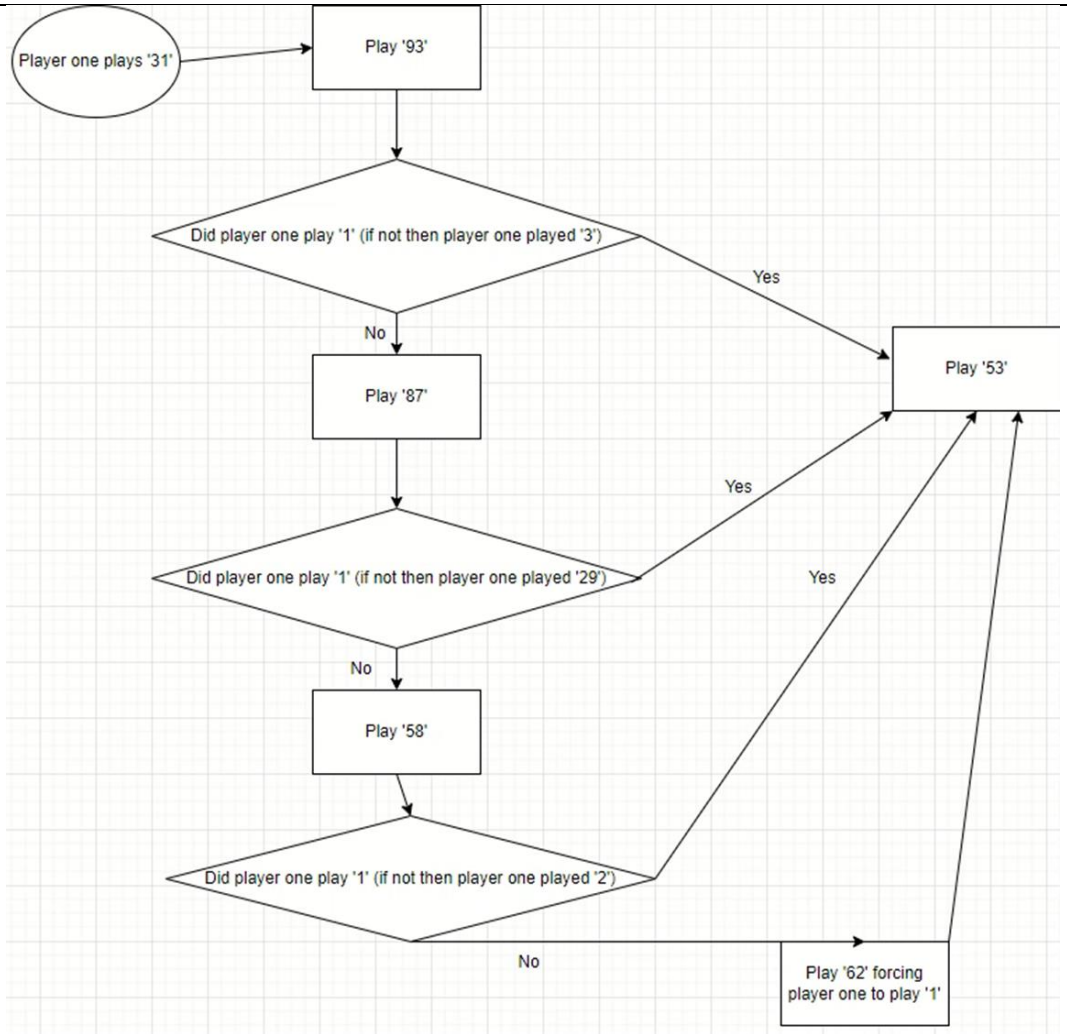
	
11	Algorithm 11: 
13	Algorithm 13: 

17	 Player one plays '17' Play '51' Did player one play '1' (if not then player one played '3') Yes No Go to Algorithm 3 Play '53'
19	 Player one plays '19' Play '57' Did player one play '1' (if not then player one played '3') Yes No Go to Algorithm 3 Play '53'
23	 Player one plays '23' Play '69' Did player one play '1' (if not then player one played '3') Yes No Go to Algorithm 3 Play '53'

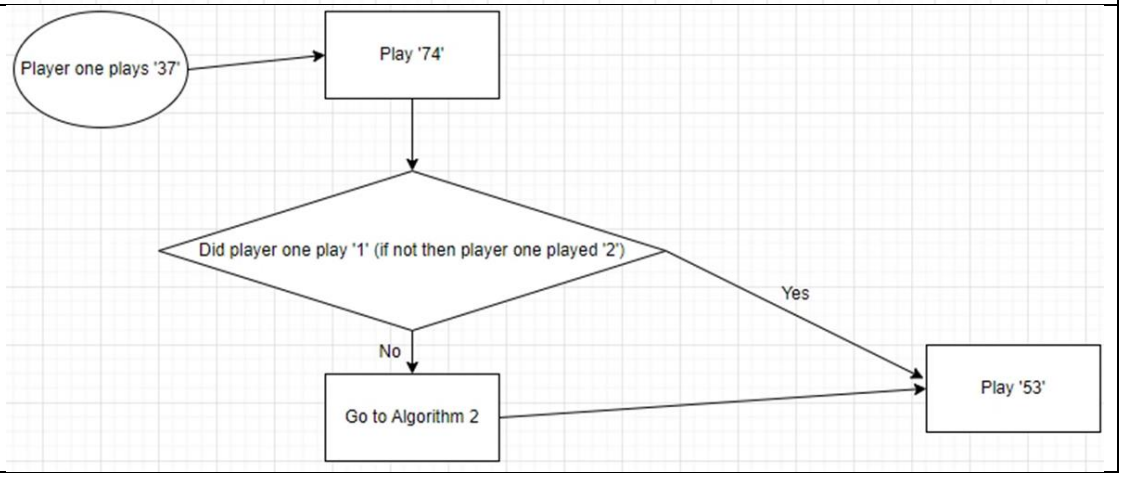
29

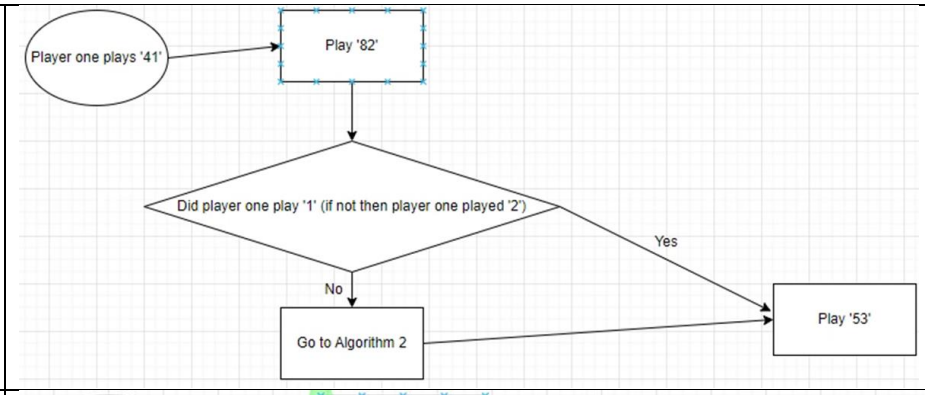
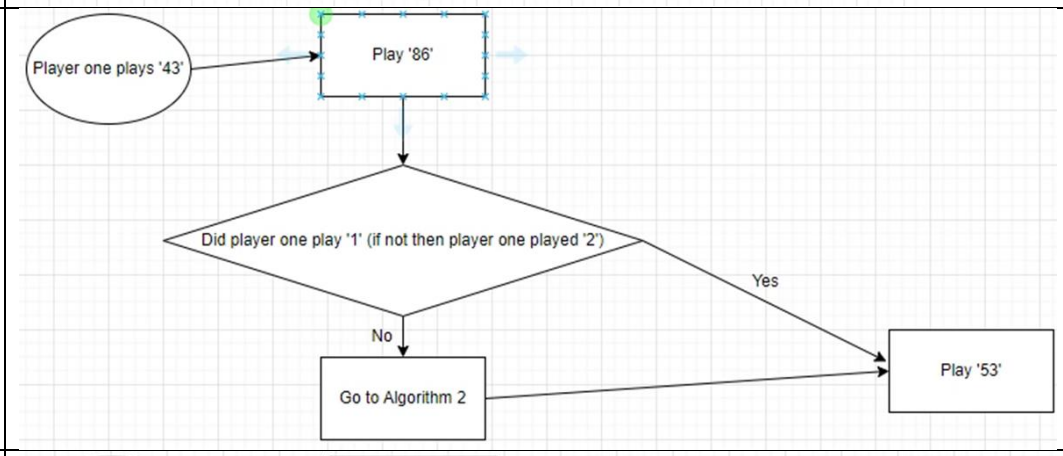
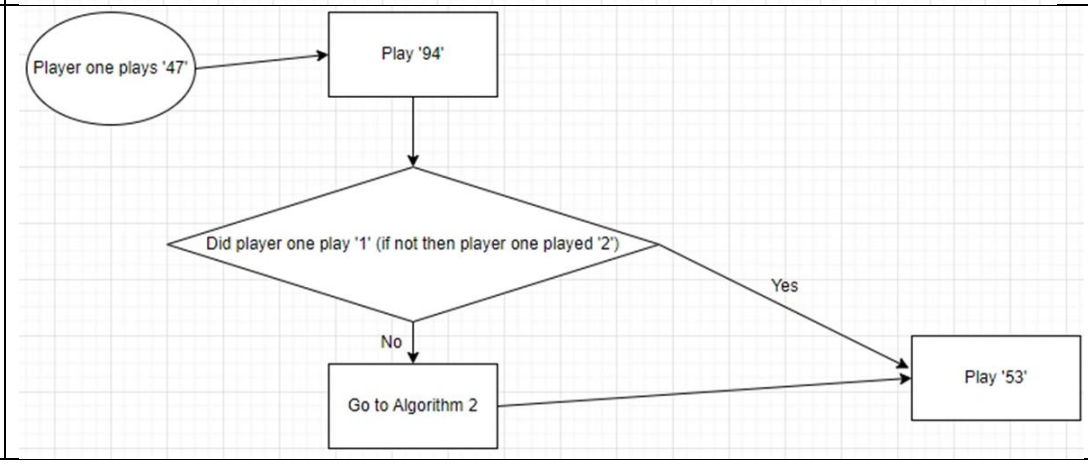

Player one plays '29'
Play '87'
Did player one play '1' (if not then player one played '3')
Yes
No
Play '53'
Play '93'
Yes
Did player one play '1' (if not then player one played '31')
Yes
No
Play '62'
Did player one play '1' (if not then player one played '2')
No
Play '58' forcing player one to play '1'

31


Player one plays '31'
Play '93'
Did player one play '1' (if not then player one played '3')
Yes
No
Play '87'
Play '53'
Yes
Did player one play '1' (if not then player one played '29')
Yes
No
Play '58'
Did player one play '1' (if not then player one played '2')
No
Play '62' forcing player one to play '1'

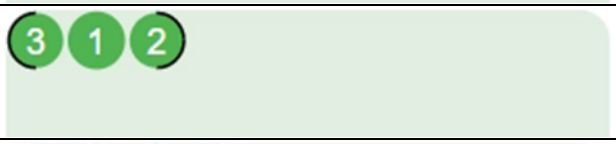
37

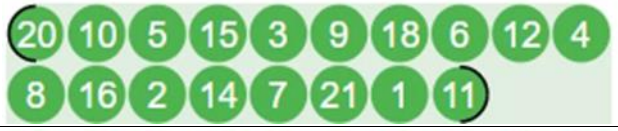
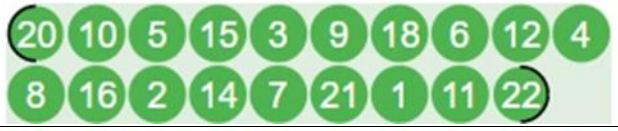
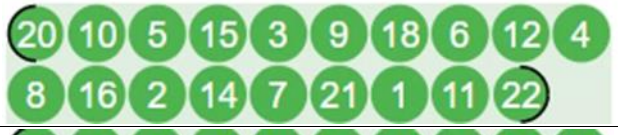

Player one plays '37'
Play '74'
Did player one play '1' (if not then player one played '2')
Yes
No
Go to Algorithm 2
Play '53'

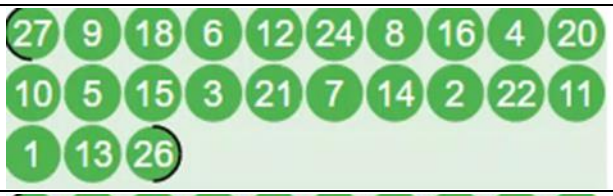
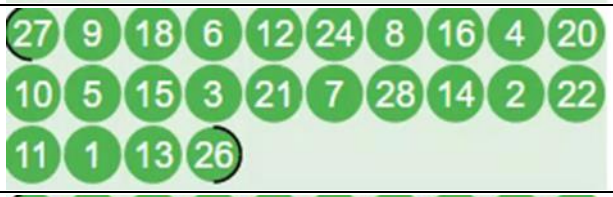
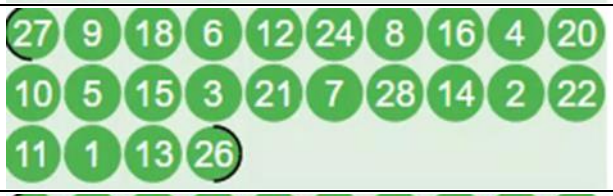
41	
43	
47	

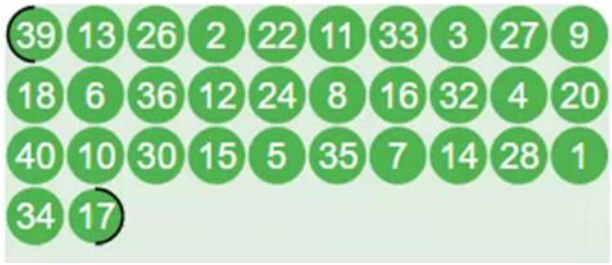
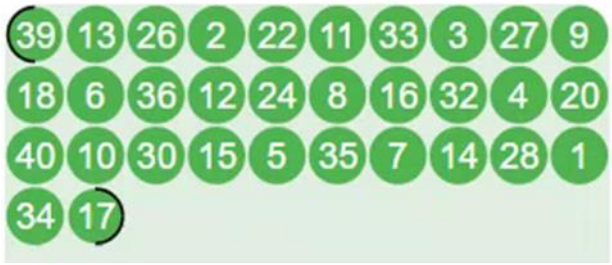
50+ factors multiples games solved by hand (couldn't find a quick simple algorithm to find these chains, took me loads of trial and error):

n	**Longest chain length**	**Example chain**	**Forbidden numbers which cannot be in a optimised chain** (there may be more than some listed)
1	1	1	None

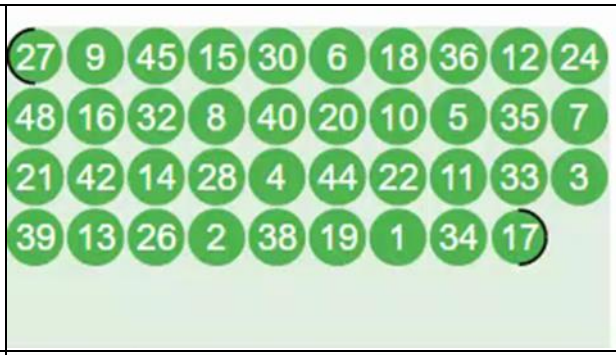
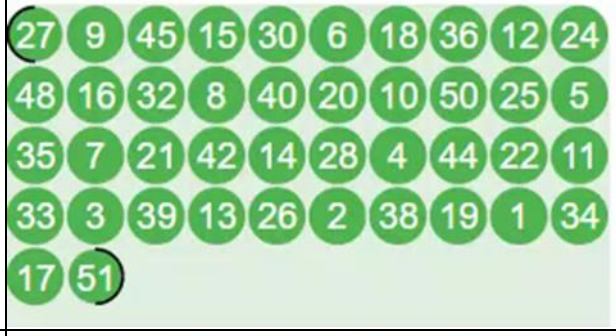
2	2		None
3	3		None
4	4		None
5	4		None
6	6		None
7	6		None
8	7		None
9	8		None
10	9		7
11	9		7,11
12	11		None
13	11		None
14	12		11, 13

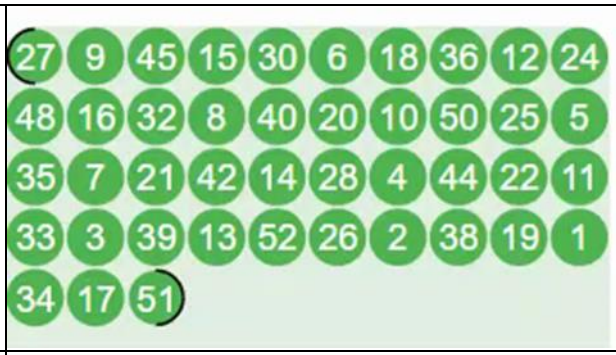
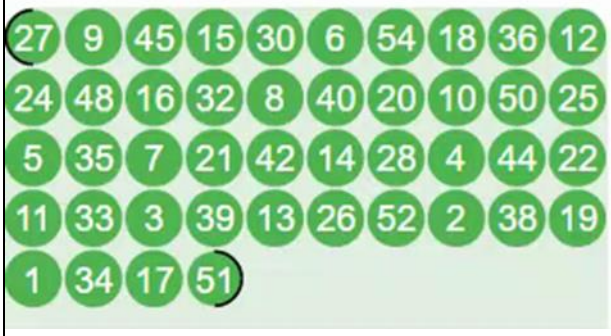
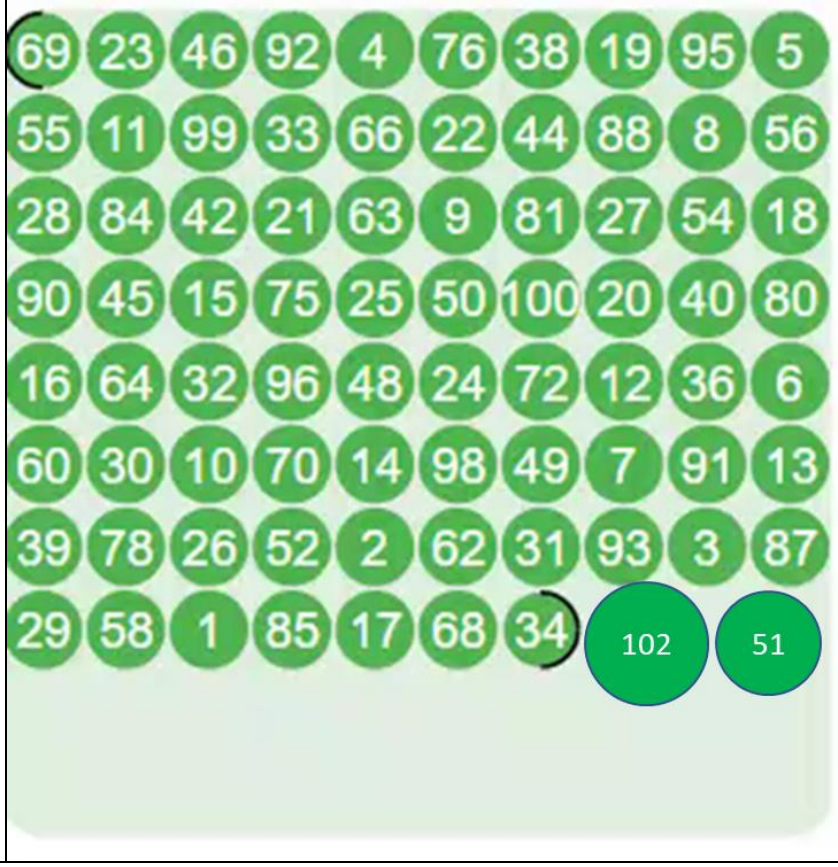
15	13		11, 13
16	14		11,13
17	14		11,13,17
18	16		None
19	16		None
20	17		None
21	18		None
22	19		13,17,19
23	19		13,17,19,23
24	21		None
25	21		None
26	22		17, 19, 23

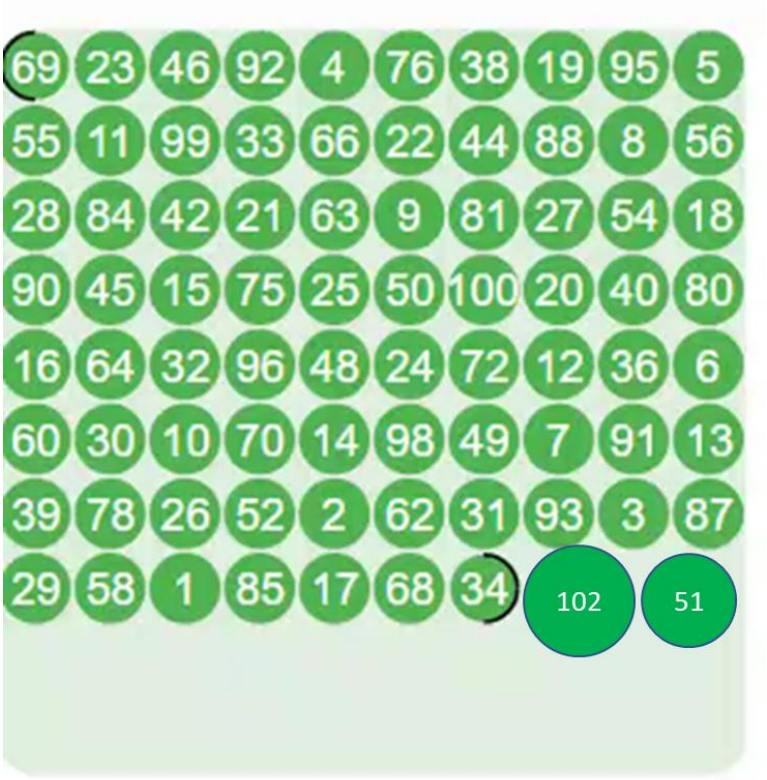
27	23		17,19,23
28	24		17,19,23
29	24		17,19,23,29
30	26		(To be done later…)
31	26		
32	27		
33	28		
34	28		
35	29		

36	30		
37	30		
38	30		
39	31		
40	32		
41	32		
42	34		

43	34		
44	36		
45	37		
46	37		
47	37		

48	39		
49	39		
50	41		
51	42		
52	43		

53	43		
54	44	 [Haven't yet solved chains length 55 to 99, chains of length 100 and 101 have already been solved]	
102	79		

103	79	 [This is the longest optimised chain I could find so far, copying chain structures from here (https://nrich.maths.org/factorsandmultiples/solution) then squeezing in a couple of numbers].	