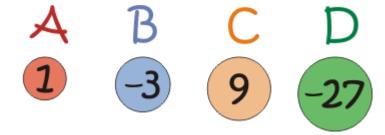## Nrich maths solution- weights

Imagine you have **two of each** of the 'weights' above.
Different combinations of the weights available allow you to make different totals.

Here are some examples:
B+C=6
B+2C=15
A+2B+C=4
2A+B+2C+D= −10

The largest total you can make is 20 (check you agree).

The smallest total you can make is $−60$ (again, check you agree).

Can you make all the numbers in between?
Can you show us how?

There are 8 different totals that can be made just using weights A and B:

2A= 2

A= 1

2A+B= -1

A+B= -2

B= -3

2A+2B= -4

A+2B= -5

2B= -6

This shows that, any number produced using weights C/D, the 2 numbers after it and the 6 numbers before it can also be produced by using weights A/B- meaning 9 different numbers can be produced with just one sum produced by the weights C/D.

As 9 different totals using C and D can be made…:

2D= -54

C+2D= -45

2C+2D= -36

D= -27

C+D= -18

2C+D= -9

0=0

C= 9

C+D= 18

…. This shows that all the numbers between -60 and 20 can be made as 9 (the no. of sums that can be produced using one C/D sum) x 9 (the number of C/D sums that can be made) is 81. The difference between -60 and 20 (inclusive) is 81.

I started using 2D. Using 2D and different sums of weights A/B, the numbers from -60 (-54-6) to -52 (-54+2) can be produced. Then, using C+2D and different sums of weights A/B, all the numbers from -51 (-45-6) to -43 can be produced and so on…

It is possible to be able to tell that all the numbers between -60 and 20 can be made without having to check it as the different sums of the weights C/D all have a difference of 9.

Is there always a unique way of producing a total, or can different combinations produce the same total?

I know that there has to be a unique way of producing a total as only 81 different sums can actually be made using the weights A/B/C/D- and, as there are 81 numbers between -60 and 20 (inclusive), the totals will all be different, none of the totals will be able to have 2 different sums of the weights to produce it.