P1. “3 = 0”

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Our two hypotheses are the following:

$$x \in \mathbf{C} \tag{1}$$

$$x^2 + x + 1 = 0. \tag{2}$$

In the dodgy proof, we have the following additional statements:

$$\begin{align}
x^2 &= -x - 1 \tag{3}\\
x &= -1 - \frac{1}{x} \tag{4}\\
x^2 + \left(-1 - \frac{1}{x}\right) + 1 &= 0 \tag{5}\\
x^2 &= \frac{1}{x} \tag{6}\\
x^3 &= 1 \tag{7}\\
x &= 1 \tag{8}\\
1 + 1 + 1 &= 0 \tag{9}\\
3 &= 0 \tag{10}
\end{align}$$

The structure of the dodgy proof is that

$$\begin{align}
(2) &\Rightarrow (3) \tag{11}\\
(3) &\Rightarrow (4) & [x \neq 0] \tag{12}\\
(4) \wedge (2) &\Rightarrow (5) \tag{13}\\
(5) &\Rightarrow (6) \tag{14}\\
(6) &\Rightarrow (7) \tag{15}\\
(7) &\Rightarrow (8) \tag{16}\\
(8) \wedge (2) &\Rightarrow (9) \tag{17}\\
(9) &\Rightarrow (10) \tag{18}
\end{align}$$

If we accept that $x \in \mathbf{C}$, then the dodgy proof is not correct, because (7) does not imply (8). (There are three complex numbers whose cube is 1, namely 1, cis $\frac{\tau}{3}$ and cis $\frac{2\tau}{3}$. [$\tau = 2\pi$.])

If we do not accept (1), but instead assume that $x \in \mathbf{R}$, then (11) through to (18) are true, but the proof is not correct, because (2) is impossible. (The discriminant of (2) is negative.)