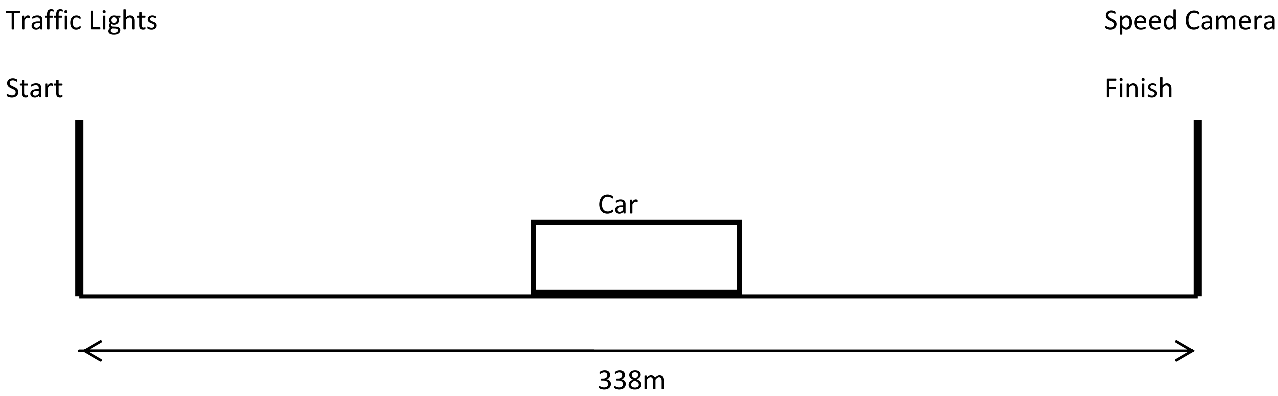
Dangerous Driver



Prosecution claims of car's velocity:

$0kmh^{-1} = 0ms^{-1}$ $133kmh^{-1} \approx 36.9ms^{-1}$

Assumptions about the road:

- It is perfectly straight
- Zero gradient change
- No speedbumps or obstacles in the road
- Uniform so that coefficient of friction constant

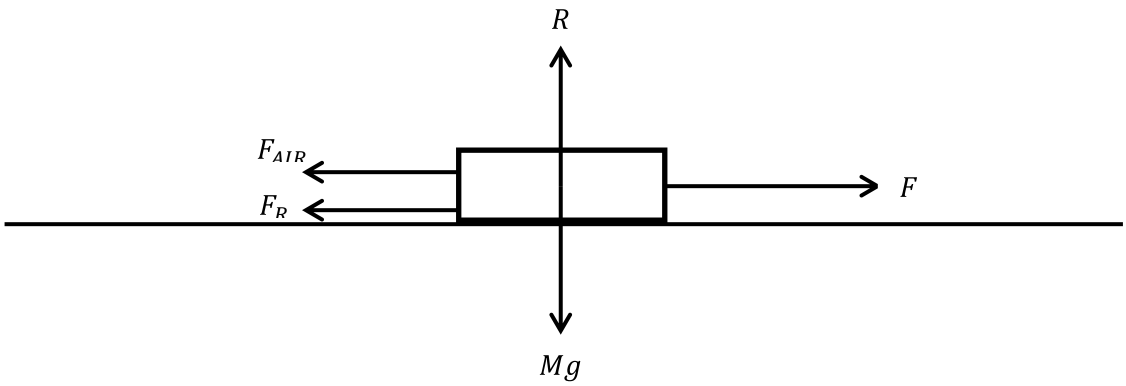
Other assumptions:

- Only the driver was in car
- Driver drove in a straight line
- Driver drove at maximum acceleration from the start of the traffic lights
- Tyres are all in excellent condition so that frictional forces constant

The information about the car states that it can accelerate from $0kmh^{-1}$ to $96kmh^{-1}$ in 10.5s, approximately from $0ms^{-1}$ to $26.7ms^{-1}$ so the maximum acceleration:

$$a_{max} = \frac{26.7 - 0}{10.5} = 2.54\ ms^{-2}$$

Looking online I found that the Ford Escort Sedan 2.0L model from 1997 fulfilled this maximum acceleration requirement, so I used it's specifications to provide the other information needed.



$F_R = \mu R = \mu Mg$ $\quad\quad$ $F_{AIR} = \frac{1}{2} \cdot A \cdot C_d \cdot D \cdot v^2$

$\mu = 0.03 \quad M = (1114 + 76) = 1190kg \quad g = 9.81ms^{-2}$

$A = 1.84m^2 \quad C_d = 0.36 \quad D = 1.29kgm^{-3}$

$\therefore F_R = 350.2\ N \quad\quad F_{AIR} = 0.427v^2 \quad kgm^{-1}$

$$F = ma_{max} - F_R - F_{AIR}$$

$$F = (1190x2.54) - 350.2 - 0.427v^2$$

$$F = 2672.4 - 0.427v^2$$

$$mv\frac{dv}{dx} = 2672.4 - 0.427v^2$$

$$\int \frac{v}{2672.4 - 0.427v^2} dv = \int \frac{1}{m}\ dx$$

$$-\frac{\ln(2672.4 - 0.427v^2)}{0.854} = \frac{x}{1190} + C$$

When x=0, v=0 $\therefore C = -\frac{\ln(2672.4)}{0.854}$

$$-\frac{\ln(2672.4-0.427v^2)}{0.854}=\frac{x}{1190}-\frac{\ln(2672.4)}{0.854}$$

$$\ln(2672.4-0.427v^2)=\ln(2672.4)-\frac{61x}{85000}$$

Therefore, $v=\sqrt{\dfrac{2672.4-e^{\left(\ln(2672.4)-\frac{61x}{85000}\right)}}{0.427}}$

When x = 338m, v = $36.7ms^{-1}\approx 132kmh^{-1}$. Therefore, assuming speed limit of 70mph $\approx 113\ kmh^{-1}$, the defendant was above the speed limit and there is no mathematical ground for rejecting the penalty.

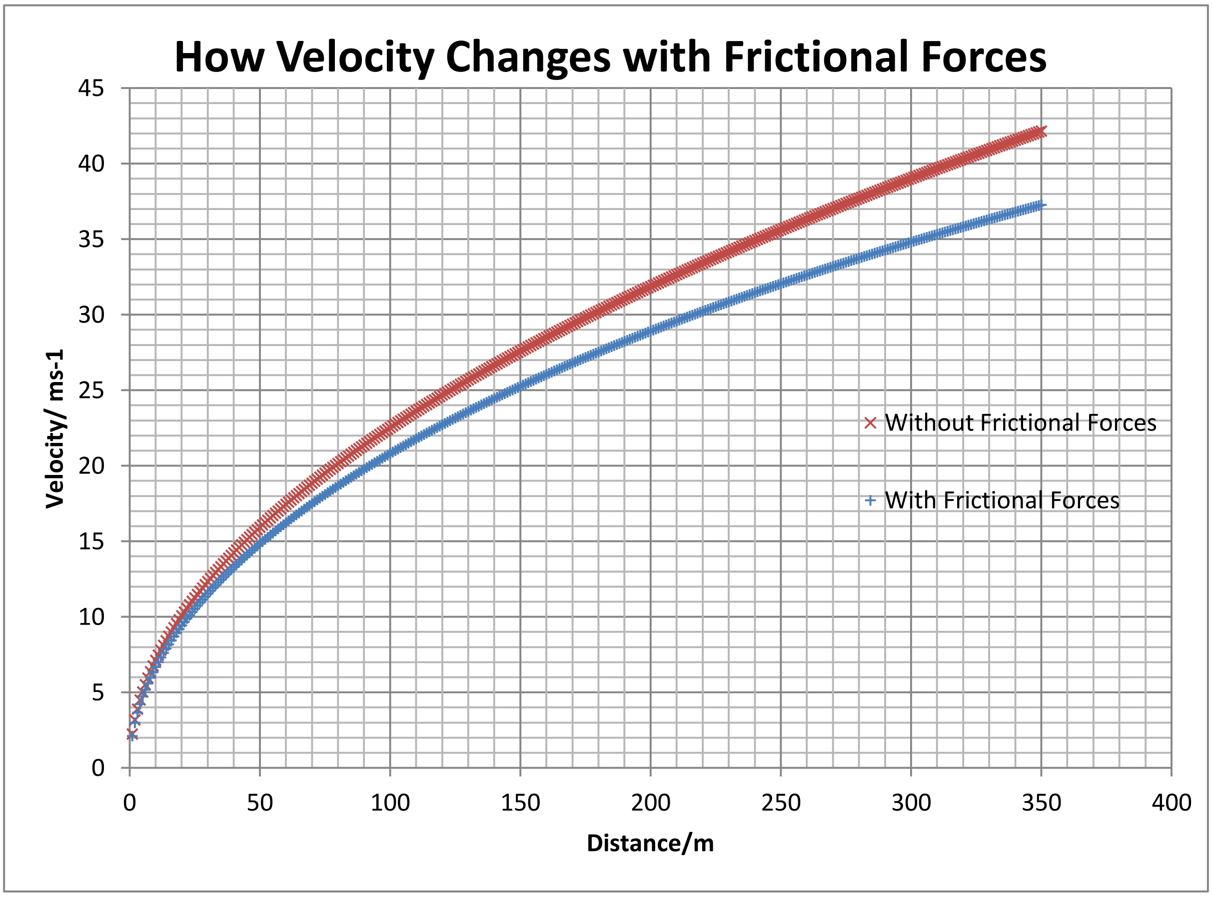
Assuming no resistive forces and a constant acceleration, the result is somewhat different.

$v^2=u^2+2as$

Where $s=338$ and $u=0$

$\therefore\ v=\sqrt{2\times 2.54\times 388}\quad v=41.4ms^{-1}$ or $149kmh^{-1}$

Now that resistive forces have been factored in, there is less doubt the driver is guilty.



As can be seen from the graph above, around $10ms^{-1}$the two graphs separate, showing that frictional forces can no longer be ignored above this threshold. Hence, air resistance and frictional forces from the road must be taken into account to properly answer the problem.