Consider two touching ford circles having centres $\left(\frac{a}{c},\frac{1}{2c^2}\right)$, $\left(\frac{b}{d},\frac{1}{2d^2}\right)$ and radii $\frac{1}{2c^2},\frac{1}{2d^2}$

respectively

If both circles are touching then the sum of radius of each circle must equal to the length between their centres.

$$\left(\frac{a}{c}-\frac{b}{d}\right)^2+\left(\frac{1}{2c^2}-\frac{1}{2d^2}\right)^2=\left(\frac{1}{2c^2}+\frac{1}{2d^2}\right)^2$$

$$\left(\frac{a}{c}-\frac{b}{d}\right)^2=\left(\frac{1}{2c^2}+\frac{1}{2d^2}\right)^2-\left(\frac{1}{2c^2}-\frac{1}{2d^2}\right)^2$$

$$\left(\frac{a}{c}-\frac{b}{d}\right)^2=\left(\frac{1}{2c^2}+\frac{1}{2d^2}+\frac{1}{2c^2}-\frac{1}{2d^2}\right)\left(\frac{1}{2c^2}+\frac{1}{2d^2}-\frac{1}{2c^2}+\frac{1}{2d^2}\right)$$

$$=\left(\frac{1}{cd}\right)^2$$

by square rooting

$$\left|\frac{a}{c}-\frac{b}{d}\right|=\left|\frac{1}{cd}\right|$$

$$\frac{|ad-bc|}{|cd|}=\frac{1}{|cd|}$$

$$|ad-bc|=1$$

Lets assume that there exist a circle with centre $\left(\frac{p}{q},\frac{1}{2q^2}\right)$ and radius $\frac{1}{2q^2}$

Which is tangent to both circles we previously mentioned.

For our convenient lets assume $\frac{a}{c}\langle\frac{b}{d}$ .since the third circle exist between above two circles we can say

$$\frac{a}{c}\langle\frac{p}{q}\langle\frac{b}{d}$$

$bq > pd \qquad pc > aq$

Considering two circles at a time (one of previously mentioned and the new circle), and using the above logic that if two circles touch each then sum of their radius should equal to length between their centres.we can state.

$$|bq - pd| = 1$$
$$bq - pd = 1 \qquad (\because \ bq > pd)$$

$$|aq - pc| = 1$$
$$pc - aq = 1 \qquad (\because \ pc > aq)$$

We can see that

$$bq - pd = pc - aq$$

rearranging

$$q(a+b) = p(c+d)$$

$$\left(\frac{p}{q}\right) = \left(\frac{a+b}{c+d}\right)$$

As mentioned in problem a, c and b,d are coprime pairs

So we can say both $a+b$ and c+d are coprimes

Then $q = c + d$

So we found a new ford circle with a centre $\left\{\left(\frac{a+b}{c+d}\right), \left(\frac{1}{2(c+d)^2}\right)\right\}$ and radius $\frac{1}{2(c+d)^2}$