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X-Dice

An X-Die "A" numbered {1, 1, 1, 1, 1, 16} is *worse* than an ordinary die N {1, 2, 3, 4, 5, 6} because:

P(A>N) = P(A=16) = $\frac{1}{6}$ and P(A<N) = P(A=1) x P(N>1) = $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

Since $\frac{25}{36} > \frac{1}{6}$, P(A>N) < P(A<N) and by definition, die A is *worse* than ordinary die N.

If we have an X-Die "P" numbered {*a*,*b*,*c*,*d*,*e*,*f*} such that *a*, *b* , *c*, *d*, *e* and *f* range from 1-6 and *a*+*b*+*c*+*d*+*e*+*f* = 21, the probability that it will show a number greater than an ordinary die N is given by the following (as $\frac{a-1}{6}$ denotes the probability that *a* is greater than the figure rolled on the ordinary die):

$$P(P>N) = \frac{1}{6}\times\frac{a-1}{6} + \frac{1}{6}\times\frac{b-1}{6} + \frac{1}{6}\times\frac{c-1}{6} + \frac{1}{6}\times\frac{d-1}{6} + \frac{1}{6}\times\frac{e-1}{6} + \frac{1}{6}\times\frac{f-1}{6}$$

$$P(P>N) = \frac{a+b+c+d+e+f-6}{36}$$

$$P(P>N) = \frac{21-6}{36} = \frac{15}{36}$$

Therefore, no matter what values one assigns for the sides of the die (i.e. *a*, *b*, *c*, *d*, *e*, and *f*), the probability that the figure displayed on a single throw will be greater than that on a normal die is equal.

We also know (as $\frac{6-a}{6}$ denotes the probability that *a* is less than the figure rolled on the ordinary die) that:

$$P(P<N) = \frac{1}{6}\times\frac{6-a}{6} + \frac{1}{6}\times\frac{6-b}{6} + \frac{1}{6}\times\frac{6-c}{6} + \frac{1}{6}\times\frac{6-d}{6} + \frac{1}{6}\times\frac{6-e}{6} + \frac{1}{6}\times\frac{6-f}{6}$$

$$P(P<N) = \frac{36-(a+b+c+d+e+f)}{36}$$

$$P(P<N) = \frac{36-21}{36} = \frac{15}{36}$$

Therefore since for an X-Die "P" numbered {*a*,*b*,*c*,*d*,*e*,*f*} such that *a*, *b* , *c*, *d*, *e* and *f* range from 1-6 and *a*+*b*+*c*+*d*+*e*+*f* = 21, P(P>N) = P(P<N) for any P: {*a*,*b*,*c*,*d*,*e*,*f*}, we can conclude that one cannot create an X-die that is better or worse than the ordinary die N.

We can also see that since $\frac{a-1}{6}$ and $\frac{6-a}{6}$ are defined as probabilities, they cannot exceed 1 or go below 0 implying that to be "efficient" with labeling in comparison to the ordinary die, *a* must fall between 1 and 6. Therefore, a "worse" die is one that inefficiently has one side *a* labeled such that $a \geq 7$, making the above X-die "A", the most "inefficient" with its side labeled 16 and thus, the worst die. The best die on the other hand, is any die in the form of X-Die "P", where all its sides are labeled with figures between 1 and 6 inclusive.