Dasom Kim
9Math Extended

Perfect Eclipse

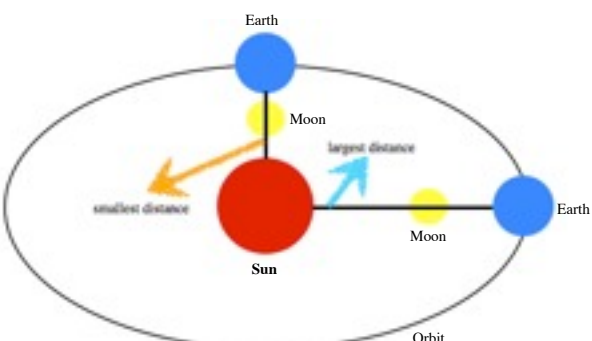
Radius of the sun: 695,500km
Radius of the moon: 1,737km
Radius of the earth: 6,371km

Smallest distance between Earth and Sun: 147,098,074km
Largest distance between Earth and Sun: 152,097,701km
Smallest distance between Moon and Earth: 356,375km
Largest distance between Moon and Earth: 406,720km

Here are some issues to consider:

- How much does the apparent size of the moon change, as its distance from the Earth changes?
- How much does the apparent size of the sun change?
- Can there ever be a perfect eclipse?

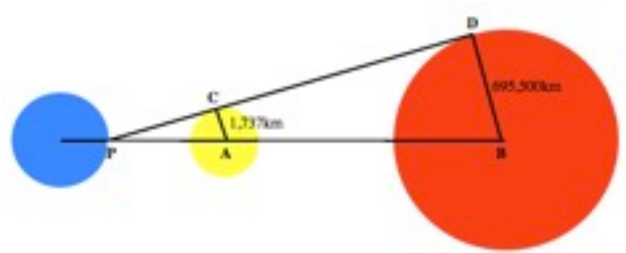
The Earth revolves round the Sun ovally. Like the picture below, while the Earth is revolving the Sun, the distances between the Sun and Earth, Earth and Moon should be different. It affects how much the Moon's shadow can cover the Sun which means it decides whether it is perfect eclipse or not.



Dasom Kim
9Math Extended

<Method 1>

- Smallest distance



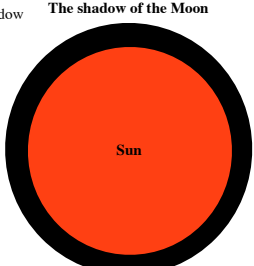
PA = (smallest distance between Earth and Moon) + (Radius of the Moon)
= 356,375km + 1,737km
= 358,112km
PB = (smallest distance between Earth and Sun) + (Radius of the Sun)
= 147,098,074km + 695,500km
= 147,793,574km

The radius of Moon's shadow reached Sun
PA : PB = 1,737 : x
358,112 : 147,793,574 = 1,737 : x
358,112x = 147,793,574 * 1,737
x = 716,863.5436...km

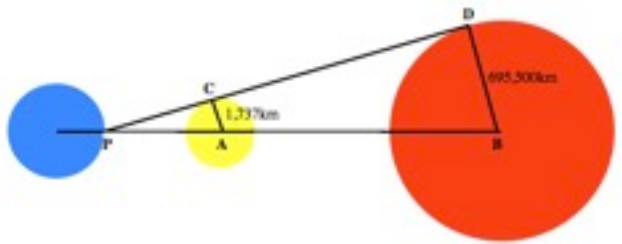
Compare between radius of Sun and radius of Moon's shadow
695,500km < 716,863.5436...km
-> the radius of Sun < the radius of Moon's shadow

-> The shadow of the Moon blocks Sun's light perfectly.

The shadow of the Moon



Dasom Kim
9Math Extended

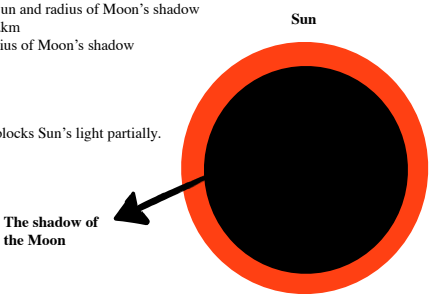


- Largest distance
PA = (largest distance between Earth and Moon) + (Radius of the Moon)
= 406,720km + 1,737km
= 408,457km
PB = (largest distance between Earth and Sun) + (Radius of the Sun)
= 152,097,701km + 695,500km
= 152,793,201km

The radius of Moon's shadow reached Sun
PA : PB = 1,737 : x
408,457: 152,793,201 = 1,737 : x
408,457x = 152,793,201 * 1,737
x =649,766.7812...km

Compare between radius of Sun and radius of Moon's shadow
695,500km > 649,766.7812...km
-> the radius of Sun > the radius of Moon's shadow

-> The shadow of the Moon blocks Sun's light partially.



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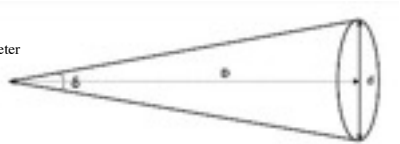
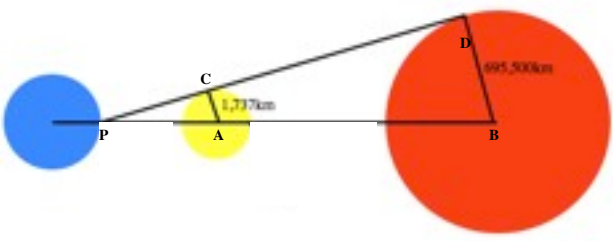
<Method 2>

Formula of calculating angular diameter

$$\delta = 2\arctan\left(\tfrac{1}{2}d/D\right)$$



- Smallest distance



D (Sun) = (smallest distance between Earth and Sun) + (Radius of the Sun)
= PB + BD
= 147,098,074km + 695,500km
= 147,793,574km
D (Moon) = (smallest distance between Earth and Moon) + (Radius of the Moon)
= PA + AC
= 356,375km + 1,737km
= 358,112km

d (Sun) = (diameter of Sun)
= (radius of Sun) x 2
= BD x 2
= 695,500 x 2
= 1,391,000km
d (Moon) = (diameter of Moon)
= (radius of Moon) x 2
= AC x 2
= 1,737 x 2
= 3,474km

Dasom Kim
9Math Extended

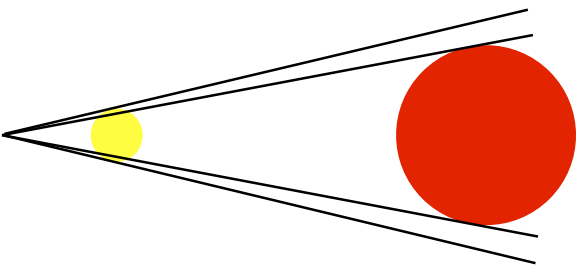
Find an angular diameter:

Sun

= 2arctan(1/2 x 1,391,000 / 147,793,574)
= 0.539251057...
= 32’35.50”

Moon

= 2arctan(1/2 x 3,474 / 358,112)
= 0.5558148766...
= 33’34.88”



-> The shadow of the Moon blocks Sun’s light perfectly.

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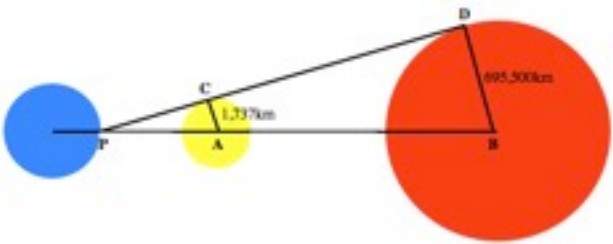
- Largest distance



D (Sun) = (largest distance between Earth and Sun) + (Radius of the Sun)
= PB + BD
= 152,097,701km + 695,500km
= 152,793,201km

D (Moon) = (largest distance between Earth and Moon) + (Radius of the Moon)
= PA + AC
= 406,720km + 1,737km
= 408,457km

d (Sun) = (diameter of Sun)
= (radius of Sun) x 2
= BD x 2
= 695,500 x 2
= 1,391,000km

d (Moon) = (diameter of Moon)
= (radius of Moon) x 2
= AC x 2
= 1,737 x 2
= 3,474km

Find an angular diameter:
Sun
θ = 2arctan(1/2 x 1,391,000 / 152,793,201)
= 0.5216061863...
= 31'29.63"
Moon
θ = 2arctan(1/2 x 3,474 / 408,457)
= 0.4873079373...
= 29'23.84"

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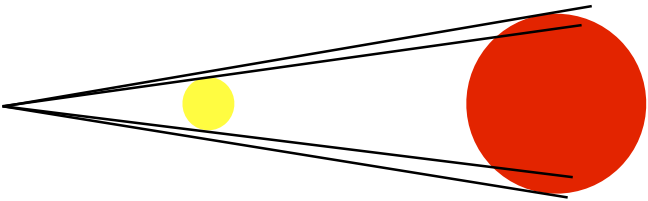

-> The shadow of the Moon blocks Sun's light partially.

The shadow of the Moon covers the Sun if those planets' positions are at the smallest distance. In other words, the perfect eclipse occurs when the moon is between the sun and the earth at the smallest distance. In contrast, if the Moon is the farthest to the Sun, the Moon's shadow cannot cover all part of the Sun which is the partial eclipse.