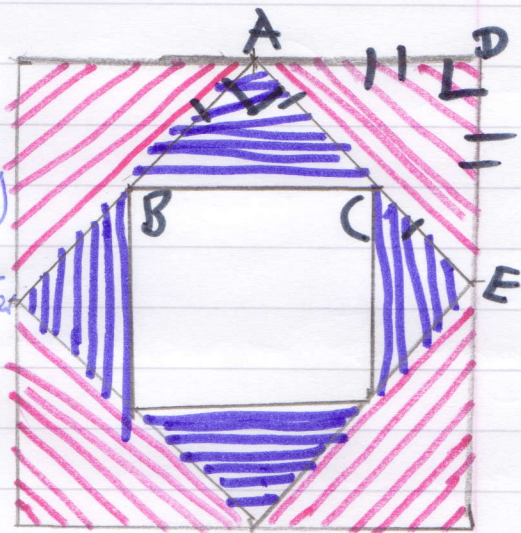


Diminishing Returns

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This is my first layer

I shall compare only two triangles (ADE and ABC) as this is a matter of symmetry.



$$AC \times 2 = AE$$

$$AE^2 = 2 \times (AD^2)$$

$$BC^2 = 2 \times (AC^2)$$

$$\text{Area of } \triangle ADE = \frac{AD^2}{2} = \frac{AE^2}{4}$$

$$\text{Area of } \triangle ABC = \frac{AC^2}{2}$$

$$\therefore \text{Proportion of } \frac{\text{pink to blue}}{\text{blue}} = \frac{\frac{AE^2}{4} \times 2}{\frac{AC^2}{2}} = 2 \quad \text{--- (1)}$$

$$\begin{aligned} \therefore \text{Proportion of blue in the first layer} &= \frac{b}{p+b} \\ &= \frac{b}{3b} \\ &= \frac{1}{3} \end{aligned}$$

If we carry on this pattern ∞ times (as in Fig. 2) the proportion of blue will be $\frac{1}{3}$.

In figure 2 the proportion of blue is easier to calculate than in figure 1.

I expect that the proportion of blue in Figure 1 is slightly less than $\frac{1}{3}$ because there is a pink square in the middle.

The proportion of blue in Figure 1 is

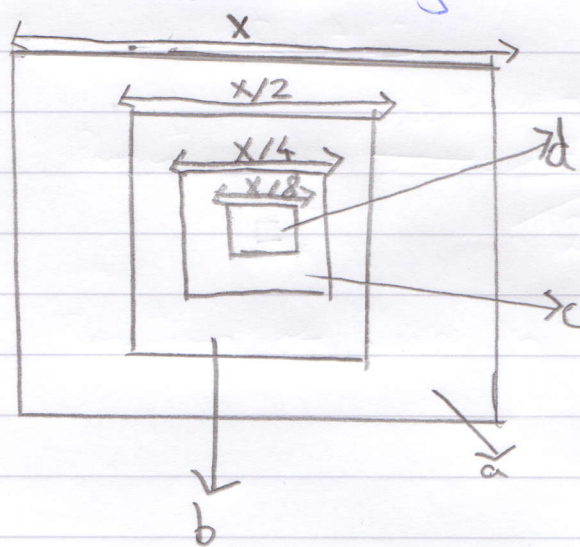
$$\frac{a + b + c}{a + b + c + d}$$

Where a = area of 1st Layer,

b = area of 2nd layer,

c = area of 3rd layer and

d = area of 4th layer.



$$a = x^2 - \left(\frac{x}{2}\right)^2 \text{ or}$$

$$x^2 - \frac{x^2}{4} \text{ or}$$

$$x^2 \left(1 - \frac{1}{4}\right) = x^2 \frac{3}{4}$$

$$b = \left(\frac{x}{2}\right)^2 - \left(\frac{x}{4}\right)^2 \text{ or}$$

$$\frac{x^2}{4} - \frac{x^2}{16} \text{ or}$$

$$\frac{x^2}{4} \left(1 - \frac{1}{4}\right) \text{ or}$$

$$\frac{x^2}{4} \cdot \frac{3}{4} \text{ or}$$

$$\frac{3x^2}{16}$$

$$c = \left(\frac{x}{4}\right)^2 - \left(\frac{x}{8}\right)^2 \text{ or}$$

$$\frac{x^2}{16} - \frac{x^2}{64} \text{ or}$$

$$\frac{x^2}{16} \left(1 - \frac{1}{4}\right) \text{ or}$$

$$\frac{x^2}{16} \cdot \frac{3}{4} \text{ or}$$

$$\frac{3x^2}{64}$$

$$d = \left(\frac{x}{8}\right)^2 \text{ or}$$

$$\frac{x^2}{64}$$

∴ Proportion of blue in figure 1 :

$$\frac{\frac{3x^2}{4} + \frac{3x^2}{16} + \frac{3x^2}{64} + \cancel{\frac{x^2}{64}}}{\frac{3}{3}} =$$
$$\frac{\frac{3x^2}{4} + \frac{3x^2}{16} + \frac{3x^2}{64} + \frac{x^2}{64}}$$

$$\frac{1 + \frac{1}{4} + \frac{1}{16}}{3 + \frac{3}{4} + \frac{3}{16} + \frac{1}{16}} = \frac{21}{64} \text{ ans}$$