Slick Summing

Charlie is working out 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ + & \underline{10} & \underline{9} & \underline{8} & \underline{7} & \underline{6} & \underline{5} & \underline{4} & \underline{3} & \underline{2} & \underline{1} \\ & 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 \end{array}$$

$$= 10 \times 11 = 110$$

So: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55

Can you see how his method works?

How could you adapt his method to work out the following sums?

$$1 + 2 + 3 + \cdots + 19 + 20$$

$$1 + 2 + 3 + \cdots + 99 + 100$$

$$40 + 41 + 42 + \cdots + 99 + 100$$

Can Charlie's method be adapted to sum sequences that don't go up in ones?

$$1 + 3 + 5 + \cdots + 17 + 19$$

$$2 + 4 + 6 + \cdots + 18 + 20$$

$$42 + 44 + 46 + \cdots + 98 + 100$$

Can you find an expression for the following sum?

$$1 + 2 + 3 + \cdots + (n-1) + n$$