## Year 7, 8, 9 Common Task: Functions and Graphs

**A first lesson**

*In the first lesson I introduce algebraic notation as a way of labelling a rule that students are already using. The motivation to look at graphing functions will come when we have a disagreement about rules.*

In silence, I write 3 →4 on the board. Underneath, I write 5 →8, and 9 →    . I hold up the board pen. Several students put hands up. I nod to one, motioning for them to take the pen. They come to the board, take the pen and write 10. In a different colour, I draw ☹

and take back the student's pen. The student returns to their seat. More hands go up - I nod to a  different student who leaves their seat and writes 12 on the board. Again, I draw ☹  and invite a third student to come to the board. They write 16. I draw ☺ and point to the space beneath the 9 (i.e. indicating that I want the student to give the next starting number). They write 6, give me the pen and return to their seat.

*By inviting the student to give the next start number, I hope to provoke an awareness that there is no pattern in these numbers - i.e. we can put anything as an input.*

I hold up the pen, and invite another student to the board. This pattern is repeated, with different students writing an answer and, if it gets a smiley face, writing a new starting number.

As more numbers are written and answered, I comment "If anyone wants to write something that might help the others, they can use the black pen on my desk and write on the right hand side of the board". A student writes:  x 2 – 2 with the black pen, another writes: x 2 – 1. A third student writes: add the number to itself and take two. As they are doing this the game with the numbers continues.

*Inviting students to write something that might help others is a strategy for supporting students in the group who may not know what is going on!*

After some time, when it seems as though most of the class could write answers, I take the pen from a student and write N as the next starting 'number'. I do not comment on what they write and keep writing N as the starting number until no on else wants to respond.

*I am aiming here to get as many different versions of the rule as possible, this can then provoke a need to work at whether the rules are the same or different.*

At this stage the board was as below:

3 → 4
5 → 8
9 → 10 ☹ 12 ☹ 16 ☺
6 → 10 ☺
4 → 6 ☺
50 → 98 ☺
49 → 96 ☺
33 → 64 ☺
101 → 200 ☺
54 → 106 ☺
99 → 196 ☺
1: N → N x 2 - 2
2: N → N x 2 - 1
3: N → N – 1 x 2
4: N → N add itself and take away 2
5: N → N + N - 2

X 2 - 2

X 2 - 1

Add the number to itself
and take two

*If there had not been differences in the rules that students wrote I would have started another game with a different rule.*

*Having generated a difference in students' responses to a task, it is now possible to work with those differences i.e. it is possible to ask questions or invite students to ask questions.*

- What do these mean? (Pointing to the rules.)
~ The first one says times your number by two and take away two.
- Can you show what you mean with one of the answers?
~ Like the four. Four times two is eight and eight take two is six.
~ On my one (x 2 - 1) I meant take away one first.
- What do you mean?
~ Four take away one is three, and three times two is six.
~ That's the same as the third rule.
~ Isn't that the same as times two and take away one?

*This question (are the rules the same or are they different?) may not come explicitly from the students, in which case I might offer the question: "Which of these rules are the same? Will these always be the same or always be different?"*

Possibilities

We might write out the rules and try substituting 10 numbers into each of them. There is a chance to focus on the conventions of writing algebra as the rules are re-written e.g. 'Mathematicians tend to write 2N instead of N x 2 but it means the same thing', or, 'You need brackets around the N - 1 if you want to show that needs to be done first'.

I might play several more function games, introducing the idea of putting the right hand number first and writing the arrow to the left.

**Where this can go**

*At some stage in this series of lessons, I will move the task onto drawing graphs of rules. This will happen when, having played a function game, we again end up with differences in the rules the students write. This can be particularly rich if one of the rules is quadratic (e.g. I have played a game with the rule double take five, one student writes the rule as 2N - 5, and another writes NN - 5).*

- One way that mathematicians can tell if rules are the same or not is to draw graphs of them. I'm going to show you how to draw a graph of a rule. I want everyone to copy this down exactly as I do it, because every time you draw a graph I want you to do it in exactly the same way.

I write the rule on the board (in the illustrations below, the rule is 2N - 5) and underneath write 5 → , 4 → , down to -5 → . Getting answers from the class, I fill in this table and next to each line write the pair of numbers as co-ordinates. Everyone then copies a set of axes, which I draw on the board. Students come to the board to plot the co-ordinates from the table, ticking them off as they go. Students copy the points onto their axes.

*Some discussion is necessary with the negative co-ordinates. I do not labour the point about rules for multiplying negatives - I see part of this activity as setting up a situation in which students can work at making their own sense of multiplying negatives, whilst their attention is on something else (i.e. drawing the graphs). Linear rules introduce an element of self-checking - if the points do not lie in a straight line then something has gone wrong.*

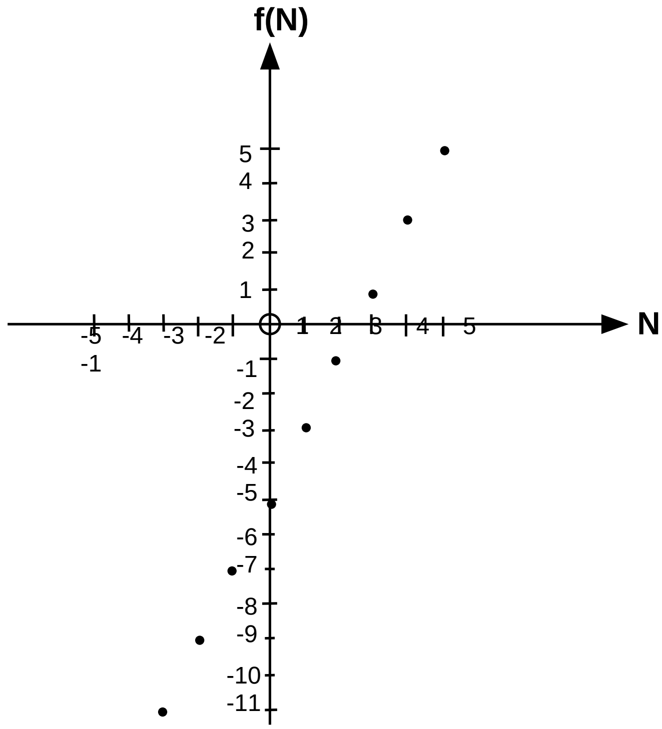
~ All the points are in a straight line.
- If you agree with that, you can join up the points using a ruler.

- Now, in your books, do a table for our other rules (e.g. (N - 5) x 2 or whatever else they have come up with) and plot the points on the same set of axes. Are the rules the same or are they different?

*The difference in the rules provides a motivation to plot the graphs. If there is a quadratic rule, this will need to be done altogether in order to discuss issues such as negative two times negative one.*

After plotting the first rule, the board might be as below:

| N | → | 2N - 5 | |
|---|---|---|---|
| 5 | → | 5 | (5,5) |
| 4 | → | 3 | (4,3) |
| 3 | → | 1 | (3,1) |
| 2 | → | -1 | (2,-1) |
| 1 | → | -3 | (1,-3) |
| 0 | → | -5 | (0,-5) |
| -1 | → | -7 | (-1,-7) |
| -2 | → | -9 | (-2,-9) |
| -3 | → | -11 | (-3,-11) |




Possibilities

Once students have begun the process of drawing graphs from rules, many possible challenges are possible. If encouraged, the students themselves will come up with questions they want to explore.

- What other rules will give the same line as this one?
- If I give you any rule, can you predict what the graph will look like without having to plot it?
- Can you be organised in the rules you try?
- Given two rules, are there any numbers that will give the same output?
- Which rules give straight lines, which rules give curved lines?

One way of structuring this work is to get students to draw their graphs on paper (not writing the table of values but including the rule, written large), which they pin up on a display board. At some stage, one or two students can be given the task of organising the graphs. When organised in some way, students can be invited to gather around the display board.

- These graphs have all be organised by a student. What does anyone else see or notice?

Categorising the graphs students have drawn and discussing classifications is a strategy for provoking questions, e.g. someone might notice all the curved graphs have NN or N2 in them, question: will this always be true?

A good lesson start part way through is to have a set of axes drawn on the board with some points drawn in a straight line and ask students what they notice and eg can they give other points on the line, can they give the equation, what about the equation of a parallel line.

Extension

- If I have a rule N->aN + b, can you describe what the graph will look like? What do a and b tell you?

- If I have a rule N->a $N^2$ + bN + c, can you describe what the graph will look like? What do a, b and c tell you?

**Guidance for notes at the end of the topic**

Reading algebra:
2n means ...
3n means ...
4n means ...
etc
$n^2$ means ...
$n^3$ means ...
$n^4$ means ...
etc
2(n+1) means ...
[Include any other bits of notation that have arisen in the topic.]
[You might also end by asking students to write down what questions remain for them at the end of this topic, which they will be able to work on next year.]
[If there have been conjectures for the equivalent of y=mx+c then write these down, as well as conjectures concerning which rules give straight lines or not.]

**Typical mathematical content**

- Reading algebra
- Substituting into formulae
- Plotting co-ordinates
- Drawing axes
- Arithmetic with negatives
- Gradients and intercepts of straight line graphs
- Recognising when a graph will be straight or not