

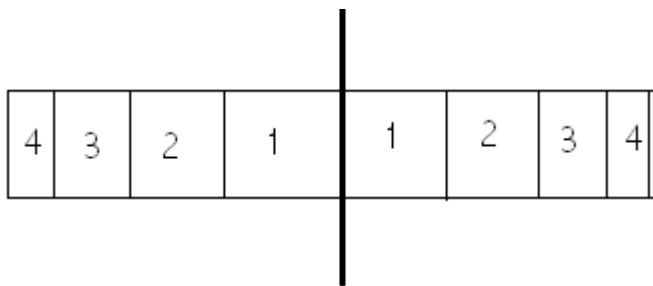
When replicating a cylinder with radius  $r$  that has  $n$  columns of squares around the outside of it then the actual width of any square on the cylinder would be the width of one column, which would be the circumference ( $2\pi r$ ) divided by the total number of columns, so it would be  $(2\pi r/n)$

The squares would appear a different width however, the width would appear to decrease the further from the middle of the cylinder the square was.

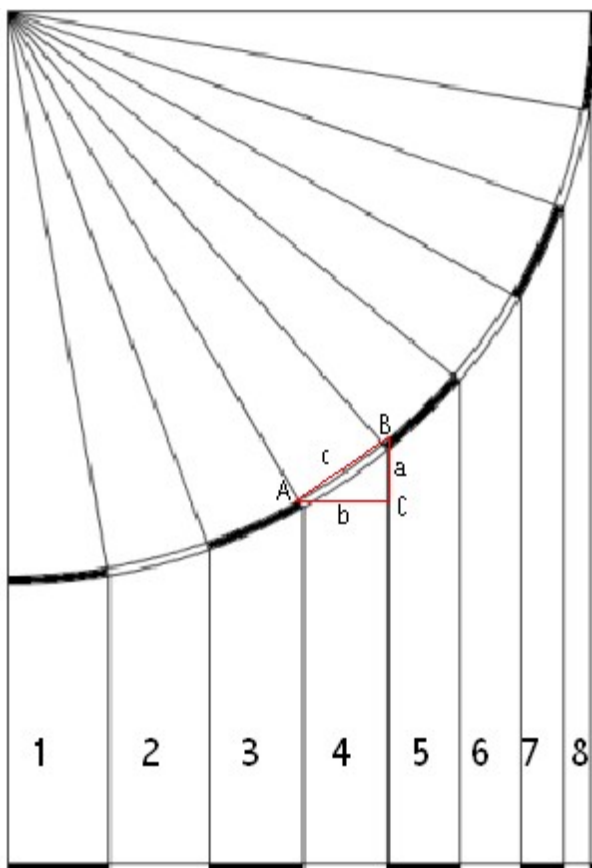
So when replicating a cylinder on a 2D surface using squares, given that you are looking directly at the 'cylinder' and that the middle of the cylinder lies in-between two columns of squares then the width of any given 'square' will be:

$$2r \sin(360/2n) \cos(360c/n - 180/n)$$

Where  $r$  is the radius of the circle,  $n$  is the number of columns of squares around the cylinder, and  $c$  is how many columns away from the middle of the cylinder the column which the square lies in is: so for example the two columns that boarder the middle of the cylinder have a  $c$  number of 1, for the next on from this  $c=2$  etc up until the number  $c/4$  (as this on either side of the middle will show half of the total columns as would be seen when viewing a real cylinder).

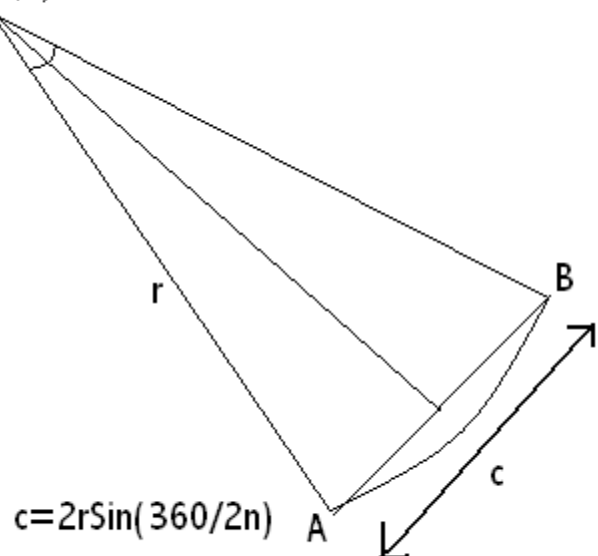


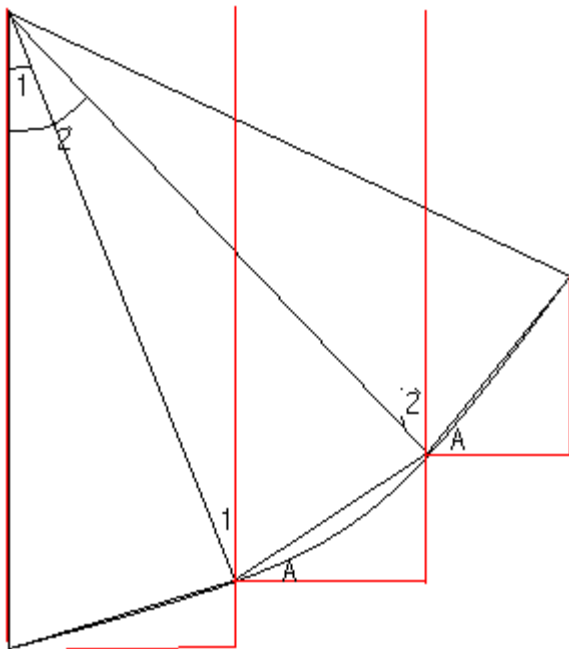
So in the inaccurate picture on the left, the line down the middle is the middle of the replicated cylinder and for each square  $c$  is the number shown.



The equation for the width is as such because to find the width of a given square, you must find the length of  $b$  in the triangle ABC shown in the diagram on the left in red. To find this you must use  $b = c \cos(A)$

Angle:  
 $(360/n)$





c is calculated by  $2r\sin(360/2n)$ , this is because each sector of the circle in picture 2 represents a column so in the triangle that c creates with the two radii shown in picture 3, the angle enclosed by the two radii will be 360 divided by total number of columns n ( $360/n$ ) and so if this triangle is split into two right angled triangles then half of c is given by  $r \sin (360/2n)$  so  $c = 2 r \sin (360/n)$ .

Therefore as  $b= c \cos A$

$$b= 2 r \sin (360/2n) \cos (A)$$

Now we must find what the value of angle A will be for any number of columns around a cylinder and for each column number.

Because of the "Z rule" concerning parallel lines and angles you get a set of equivalent angles shown by the 1 and 2 in the diagram, since  $1 = 360/n$  and  $2 = 2 \times 360/n$ , the angle going into the cth sector would be  $360c/n$ .

Using this, angle A would be given by

$$90 - ((90 - 180/n) - ((c-1)(360/n)))$$

which simplifies to give

$$180/n + (360c - 360)/n \text{ which is equal to}$$

$$360c/n - 180/n$$

You can also come to this conclusion by seeing that angle A will increase in a sequence with a common difference of  $360/n$  and a first term (c number of 1) of  $360/2n$  and therefore A can be formulated by  $360/n \times c + (360/2n - 360/n) = 360c/n - 360/2n = 360c/n - 180/n$

So if  $A = 360c/n - 180/n$   
and  $b = 2 r \sin(360/n) \cos(A)$

Then  $b = 2 r \sin(360/2n) \cos(360c/n - 180/n)$  as said at the beginning .

So this is how to replicate a cylinder in two dimensions: drawing columns of squares out from the middle of the replicated cylinder you must make the width of each column  $= 2 r \sin(360/2n) \cos(360c/n - 180/n)$  where r is the radius of the cylinder you want to replicate, n is the total number of columns around the replicated cylinder and c is the number of columns away from the middle of the cylinder. This will replicate a cylinder which has squares of actual size  $(2\pi r/n)$ .