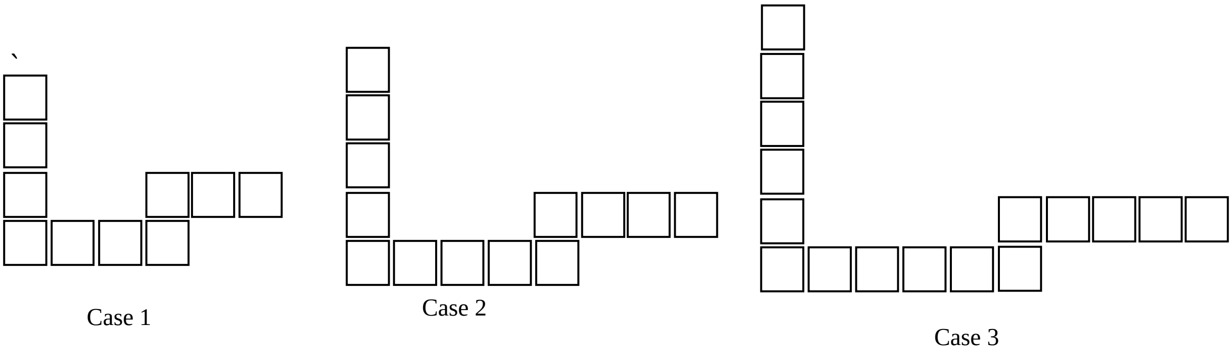
An example of a group-worthy task in Railside school.

Jo Boaler, The University of Sussex

(the following is an adapted extract from *The Elephant in the Classroom: Helping Children Learn and Love Maths,* Souvenir Press, 2009)

In one of the lessons I observed students were learning about functions. The students had been given what the teachers referred to as "pile patterns". Different students had been given different patterns to work with. Pedro was given the pattern below, which includes the first 3 cases:

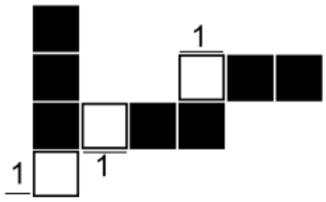


The aim of the activity was for students to work out how the pattern was growing and to represent this as an algebraic rule, a t-table, a graph and a generic pattern. Students also needed to show the 100th case in the sequence, having been given the first 3 cases.

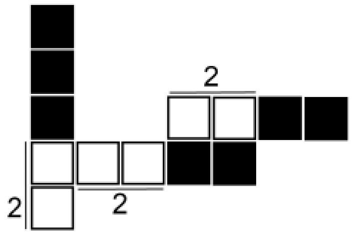
Pedro started by working out the numbers that went with the first 3 cases and he put these in his t-table:

Case number	Number of Tiles
1	10
2	13
3	16

He noted at this point that the pattern was "growing" by 3 each time. Next he tried to *see* how the pattern was growing in his shapes, and after a few minutes he saw it! He could see that each of the 3 sections grew by 1 each time. He represented the first two cases in this way:



Case 1



Case 2

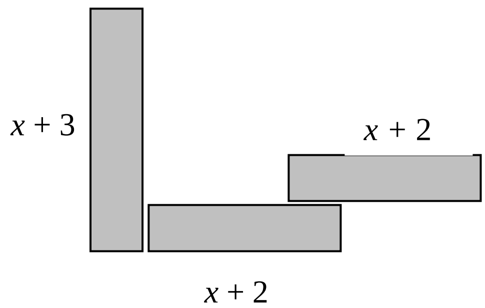
He could see that there were 7 tiles that always stayed the same and were present in the same positions (this was the way he visualized the pattern growth, but there are other ways of visualizing it). In addition to the 'constant' 7 there were tiles that grew with every case number. So, for example, if we just look at the vertical line of tiles:

Case 1

Case 2

We see that in case 1, there is 1 at the bottom, plus 3. In case 2 there is 2 plus 3. In case 3 there would be 3 plus 3 and in case 4 there would be 4 plus 3 and so on. The three is a constant but there is one more added to the lower section of tiles each time. We can also see that the growing section is the same size as the case number each time. When the case is 1 the total number of tiles is 1 plus 3, when the case is 2 the total is 2 plus 3, we can assume from this that when it is the 100th case, there will be 100 + 3 tiles. This sort of work – considering, visualizing and describing patterns is at the heart of mathematics.

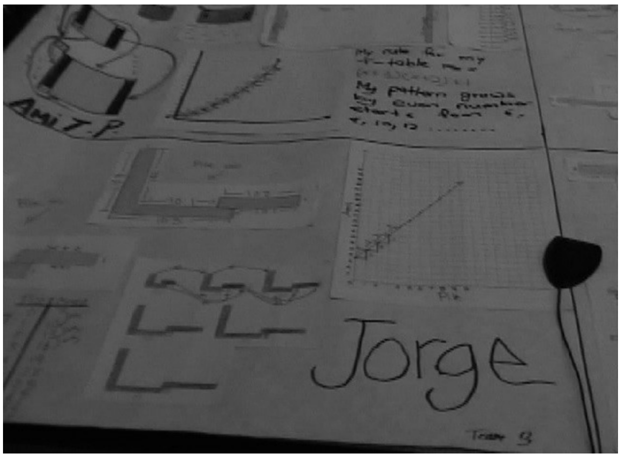
Pedro represented his pattern algebraically in the following way:



Where x stood for the case number. By adding together the three sections he could now represent the whole function as 3x+7.

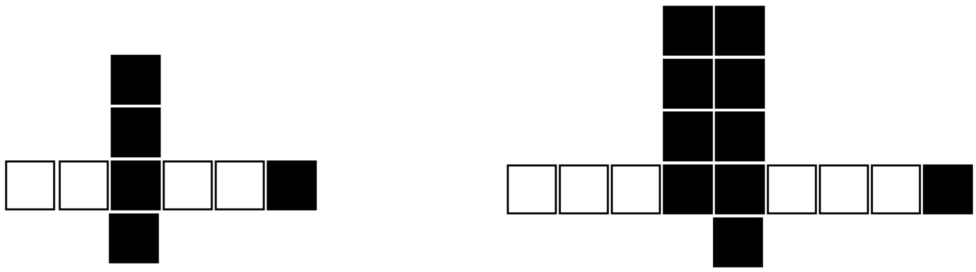
At this point I should explain something about algebra, for those who find this example totally bewildering. When a friend of mine read about this pattern she was utterly lost and I realized that her confusion came from the way she had learned algebra in her traditional maths classes. She looked at the pattern with me and saw that the student had represented it as 3x + 7 and she asked me, so what is x? I said that x was the case number, so in the first case x is 1, in the second x is 2 etc. This completely confused her and I realized that she was confused because to her x was always meant to be *a single* number. She had spent so many years of maths classes "solving for x" – rearranging equations to find out what number x was, that she, like millions of school children, had missed the most important point about algebra – that x is used to represent a *variable*. The reason that algebra is used so pervasively by mathematicians, scientists, medics, computer programmers and many other professionals is because patterns – that grow and change – are central to their work and to the world, and algebra is a key method in describing and representing them. My friend could see that the pattern increased by a different amount each time but was just not used to using algebra to represent a *changing* quantity. But the task in this problem – to find a way of visualizing and representing the pattern, using algebra to describe the changing parts of the pattern – is extremely important algebraic work. The way that most people learn algebra hides the meaning of algebra, it stops them from using it appropriately and it hinders their ability to see the usefulness of algebra as a problem-solving tool in mathematics and science.

Pedro was pleased with his work and he decided to check his algebraic expression with his t-table. Satisfied that 3x+7 worked, he set about plotting his values on a graph. I left the group as he was eagerly reaching for graph paper and coloured pencils. The next day in class I checked in with him again. He was sitting with 3 other boys and they were designing a poster to show their 4 functions. Their 4 desks were pushed together and covered by a large poster that was divided into 4 sections. From a distance the poster looked like a piece of mathematical artwork with colour-coded diagrams, arrows connecting different representations to each other and large algebraic symbols.



After a while the teacher came over and looked at the boys' work, talking with them about their diagrams, graphs, and algebraic expressions, probing their thinking to make sure they understood the algebraic relationships. He asked Pedro where the 7 (from 3x+7) was represented on his graph. Pedro showed the teacher and then decided to show the +7 in the same colour on his tile patterns, his graph and in his algebraic expression. The communication of key features of functions using colour-coding was something all students were taught in the Railside approach, to give meaning to the different representations. This helped the students learn something important – that the algebraic expression shows something tangible and that the relationships within the expression can also be seen in the tables, graphs and diagrams.

Juan, sitting at the same table, had been given a more complicated, non-linear pattern that he had colour coded in the following way.



See if you can work out how the pattern is growing and the algebraic expression that represents it!

As well as producing posters that showed linear and non-linear patterns, the students were asked to find and connect patterns, within their own pile patterns, and across all four teammates' patterns, and to show the patterns using technical writing tools. One of the aims of the lesson was to teach students to look for patterns within and among representations and to begin to understand generalization. Because some of the pile patterns were non-linear, this was a complicated task for year 9 students, and it provoked much discussion, consternation, and learning!