$$\left(2-x^2\right)^{x^2-3\sqrt{2}x+4} = 1$$

Solving for x

When x is real:

If x is real then the above equation will equal one if:

1. $2-x^2=1$
2. $x^2-3\sqrt{2}x+4=0$ where $2-x^2 \neq 0$
3. $2-x^2=-1$ and $x^2-3\sqrt{2}x+4$ is even.

Now considering each case:

1. $$\begin{aligned} 2-x^2 &= 1 \\ -x^2 &= -1 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

2. $$\begin{aligned} x^2-3\sqrt{2}x+4 &= 0 \\ \left(x-\sqrt{2}\right)\left(x-2\sqrt{2}\right) &= 0 \end{aligned}$$
   Therefore, $x=\sqrt{2}$ or $x=2\sqrt{2}$

However, when $x=\sqrt{2}$, $2-x^2=0$ so this is not a valid solution. Only $x=2\sqrt{2}$ is valid in this case,

3. $$\begin{aligned} 2-x^2 &= -1 \\ x^2 &= 3 \\ x &= \pm\sqrt{3} \end{aligned}$$

Now need to verify that $x^2-3\sqrt{2}x+4$ is even for these solutions.

When $x=\sqrt{3}$:

$$\begin{aligned} \left(\sqrt{3}\right)^2-3\sqrt{2}\left(\sqrt{3}\right)+4 &= 3-3\sqrt{6}+4 \\ &= 7-3\sqrt{6} \end{aligned}$$

Which is clearly not an even number so solution is invalid.

Similarly, when $x=-\sqrt{3}$,

$$x^2-3\sqrt{2}x+4=7+3\sqrt{6}$$

Thus, neither of these solutions are valid.

Therefore, $x=\pm 1$ or $x=2\sqrt{2}$ are all possible real solutions to the equation.

When x is allowed to be complex:

Finding the solution is made easier by factorising the original equation:

$$\left(2-x^2\right)^{x^2-3\sqrt{2}x+4}=1$$

$$\left[\left(\sqrt{2}-x\right)\left(\sqrt{2}+x\right)\right]^{x^2-3\sqrt{2}x+4}=1$$

So we now have to expressions $\left(\sqrt{2}-x\right)^{x^2-3\sqrt{2}x+4}$ and $\left(\sqrt{2}+x\right)^{x^2-3\sqrt{2}x+4}$ whose product must be 1.

If we first consider $\left(\sqrt{2}+x\right)^{x^2-3\sqrt{2}x+4}$, letting $x=a+bi$ then we get:

$$\left(\sqrt{2}+a+bi\right)^{(a+bi)^2-3\sqrt{2}(a+bi)+4}=\left(\sqrt{2}+a+bi\right)^{a^2+2abi-b^2-3\sqrt{2}a-3\sqrt{2}bi+4}$$

$$=\left(\sqrt{2}+a+bi\right)^{a^2-b^2-3\sqrt{2}a+4+\left(2ab-3\sqrt{2}b\right)i}$$

If we then express $\sqrt{2}+a+bi$ in Euler form :

$$\sqrt{2}+a+bi=r_1e^{\theta_1 i} \quad \text{with} \quad r_1=\sqrt{\left(\sqrt{2}+a\right)^2+b^2}$$

$$\text{and} \quad \theta_1=\tan^{-1}\left(\frac{b}{\sqrt{2}+a}\right)$$

So we now have:

$$r_1e^{\theta_1 i\left(a^2-b^2-3\sqrt{2}a+4+\left(2ab-3\sqrt{2}b\right)i\right)}=r_1e^{\theta_1 i\left(2ab-3\sqrt{2}b\right)i}\times e^{\theta_1 i\left(a^2-b^2-3\sqrt{2}a+4\right)}$$

$$=r_1e^{\left(3\sqrt{2}-2a\right)b\theta_1}e^{\left(a^2-b^2-3\sqrt{2}a+4\right)\theta_1 i}$$

Note that this is simply another complex number with modulus $r=r_1e^{\left(3\sqrt{2}-2a\right)b\theta_1}$ and argument $\theta=\left(a^2-b^2-3\sqrt{2}a+4\right)\theta_1$.

If we now consider $\left(\sqrt{2}-x\right)^{x^2-3\sqrt{2}x+4}$ where $x=a+bi$, following the same process as above,

$$\left(\sqrt{2}-a-bi\right)^{a^2-b^2-3\sqrt{2}a+4+\left(2ab-3\sqrt{2}b\right)i}=r_2e^{\left(3\sqrt{2}-2a\right)b\theta_2}e^{\left(a^2-b^2-3\sqrt{2}a+4\right)\theta_2 i}$$

with $r_2=\sqrt{\left(\sqrt{2}-a\right)^2+b^2}$ and $\theta_2=\tan^{-1}\left(\frac{b}{\sqrt{2}-a}\right)$

So we now have two complex numbers, which we will call z_1 and z_2:

$$z_1 = r_1 e^{(3\sqrt{2}-2a)b\theta_1} e^{(a^2-b^2-3\sqrt{2}a+4)\theta_1 i}$$
$$z_2 = r_2 e^{(3\sqrt{2}-2a)b\theta_2} e^{(a^2-b^2-3\sqrt{2}a+4)\theta_2 i}$$

that when multiplied together must give 1:

$$r_1 e^{(3\sqrt{2}-2a)b\theta_1} e^{(a^2-b^2-3\sqrt{2}a+4)\theta_1 i} \times r_2 e^{(3\sqrt{2}-2a)b\theta_2} e^{(a^2-b^2-3\sqrt{2}a+4)\theta_2 i} = 1$$
$$r_1 r_2 e^{(3\sqrt{2}-2a)b\theta_1 + (3\sqrt{2}-2a)b\theta_2} e^{(a^2-b^2-3\sqrt{2}a+4)\theta_1 i + (a^2-b^2-3\sqrt{2}a+4)\theta_2 i} = 1$$
$$r_1 r_2 e^{(3\sqrt{2}-2a)(\theta_1+\theta_2)b} e^{(a^2-b^2-3\sqrt{2}a+4)(\theta_1+\theta_2)i} = 1$$

Now, the left hand side represents a complex number with argument $(a^2 - b^2 - 3\sqrt{2}a + 4)(\theta_1 + \theta_2)$. However, for the left hand side to be 1 or in fact any positive real number this argument must be zero:

$$(a^2 - b^2 - 3\sqrt{2}a + 4)(\theta_1 + \theta_2) = 0$$
$$\theta_1 + \theta_2 = 0$$

However, both θ_1 and θ_2 are arguments of complex numbers with the same imaginary component b, such that both must be positive or both must be negative. For this reason, both must either be π or 0 to fulfil the above equation, in which case b=0.

This indicates that there are in fact no complex solutions to the equation $(2 - x^2)^{x^2 - 3\sqrt{2}x + 4} = 1$, since $x = a + 0i$ is simply $x = a$, a real number. Since all real solutions were already found above, we can now be certain that all possible solutions have been found.