This seemed to be quite easy, as I could see how the shapes would grow. Because of the rules, there are definite patterns here which are cyclical (I thought of it as a repeating 'lifecycle', or infinite evolution).

I drew a chart to record the information, and decided my headings by looking at the properties of a one by one by one cube. As I worked, I drew pictures and used two colours to help show how each shape doubled in volume, blue for the existing model, and red for the extension, then grouped my models to show the choices and path I took. What became very obvious is the cyclical repeat in the shapes generated: cube becomes cuboid, becomes larger cuboid then returns to cube; which becomes cuboid, becomes larger cuboid and again returns to cube.

$T_{1,}\ T_{4,}\ T_7, T_{10} etc$ ... are cubes. The most economical path (using the least pebbles), dictates this. You can almost imagine the 8 'unit cubes' which make up the T_4 cube merge into one enlarged cube, and just repeat the cycle. It is infinite. The drawings on the next few pages really make this clear. I have also highlighted this on the chart below.

Because I'm not that good at drawing, I drew the models in 2-d, then added the third dimension. My pictures show solid cubes and cuboids. To count the pebbles (all of them, not just the ones you can see), I thought of the pebbles as being arranged in 'planes'. I counted the pebbles in the first plane (2-d), then multiplied this by the number of layers (or planes), turning a 2-d shape into a 3-d shape. This made it very easy to visualise and calculate. I've tried to make this clear, by giving extra detail in the chart below:

Term	Shape	Volume	Pebbles	Surface Area	Exterior Pebbles	Interior Pebbles
1	Cube	1 unit^3	(4 x 2) = 8	6 units^2	8	0
2	Cuboid	2 unit^3	(6 x 2) = 12	10 units^2	12	0
3	Cuboid	4 unit^3	(9 x 2) = 18	16 units^2	18	0
4	Cube	8 unit^3	(9 x 3) = 27	24 units^2	26	1
5	Cuboid	16 unit^3	(15 x 3) = 45	40 units^2	42	3
6	Cuboid	32 unit^3	(25 x 3) = 75	64 units^2	66	9
7	Cube	64 unit^3	(25 x 5) = 125	96 units^2	98	27
8	Cuboid	128 unit^3	(45 x 5) = 225	160 units^2	162	63

Please look at the next few pages of diagrams, before reading any more.

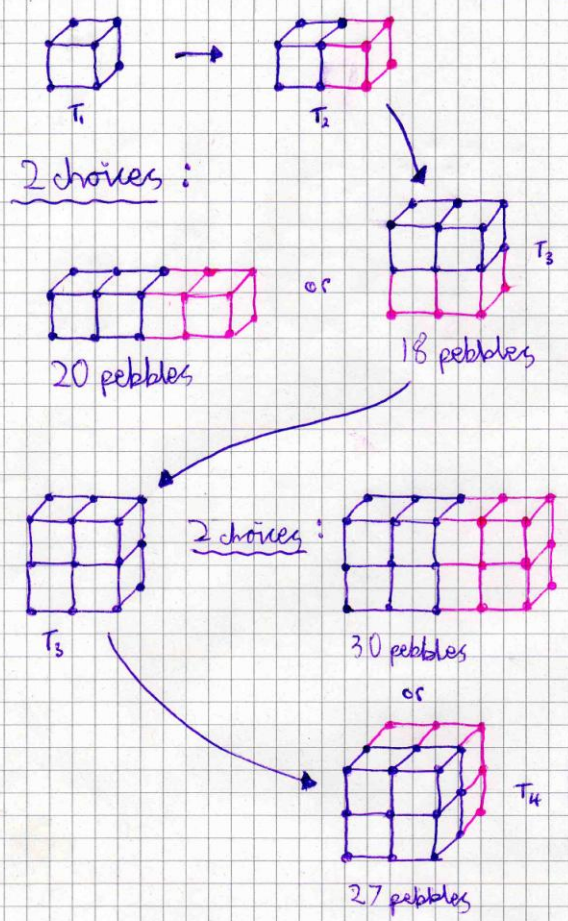
To grow from T_1 to T_2 there is only one choice (everything else is rotational symmetry).

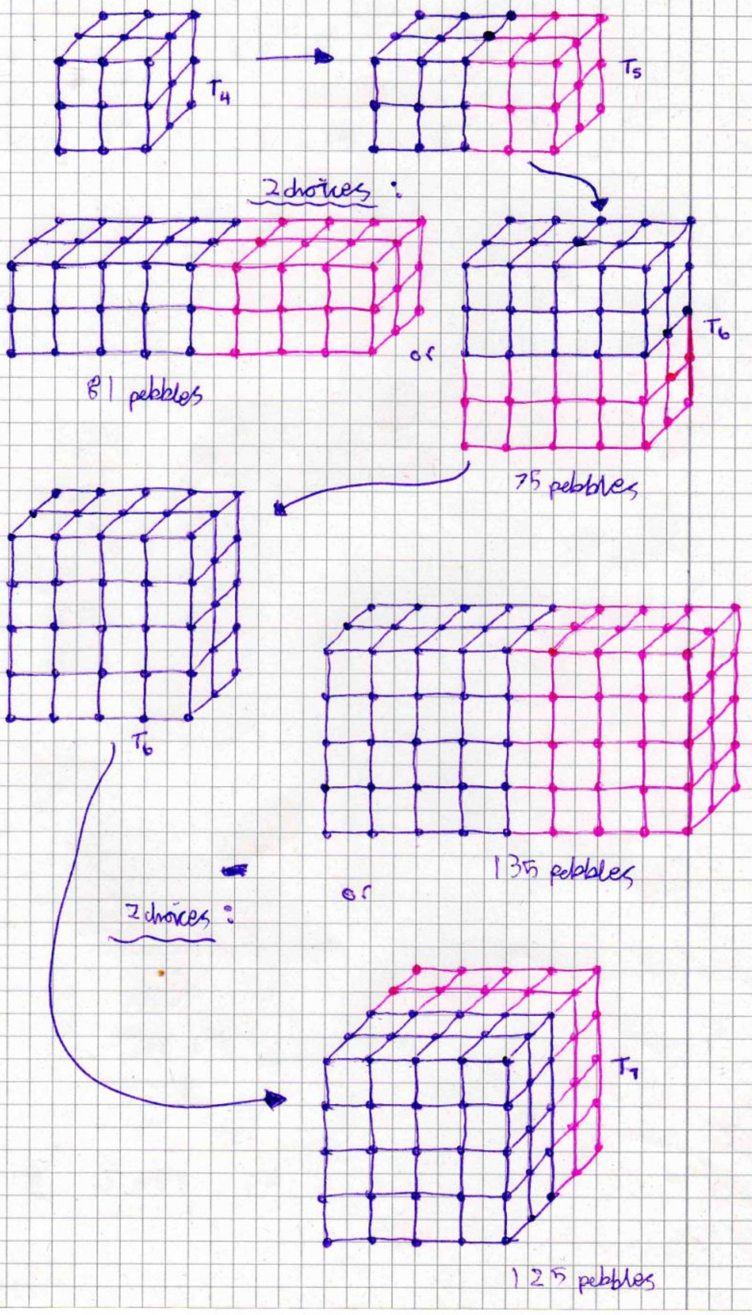
To grow from T_2 to T_3 there are two choices. The shape must grow along one plane. This plane needs to have the largest surface area possible so the number of new pebbles joining is fewest. Because T_2 is a cuboid, it's an easy choice.

To grow from T_3 to T_4 again there are two choices. Again the shape must grow along a plane with the largest surface area possible so the number of new pebbles joining is fewest. T_3 is a cuboid, so again, it's not difficult.

The journey from $T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4$ forms a pattern, that is repeated in $T_4 \rightarrow T_5 \rightarrow T_6 \rightarrow T_7$ and so on, infinitely.

Each cycle can itself be thought of as a term in the evolution of the shape, which I used in my expressions.


T1
T2
2 choices :
or
T3
20 pebbles
18 pebbles
T3
2 choices :
30 pebbles
or
T4
27 pebbles



T4
T5
2 choices :
81 pebbles
or
T6
75 pebbles
T6
135 pebbles
or
2 choices :
T7
125 pebbles

Initially I thought that the number of pebbles: 8, 12, 18, 27, 45, 75, 125, 225, was recursive, a bit like Fibonacci numbers, with $P_1 = 8, P_2 = 12$, etc.. so my first thoughts were $P_n = (2 \times P_{(n-1)}) - \left(\frac{P_{(n-1)}}{2}\right)$, but this only held for the first few terms:

$$P_1 = 8, \quad P_2 = (2 \times 8) - \left(\frac{8}{2}\right) = 12, \quad P_3 = (2 \times 12) - \left(\frac{12}{2}\right) = 18, \quad P_4 = (2 \times 18) - \left(\frac{18}{2}\right) = 27$$

But $P_5 = (2 \times 27) - \frac{27}{2} \neq$ integer, so I wondered if I should modify my formula to:

$$P_n = (2 \times P_{(n-1)}) - \left(\frac{P_{(n-1)}}{L}\right)$$, where L = number of layers (rather than planes to save confusion).

$P_5 = (2 \times 27) - \frac{27}{3} = 45$ worked, as did $P_6 = (2 \times 45) - \frac{45}{3} = 75$, $P_7 = (2 \times 75) - \frac{75}{5} \neq 125$. And when I went back over the first few terms $P_4 = (2 \times 18) - \frac{18}{3} \neq 27$ would not have worked either.

I needed to think a little deeper. The cycle of shapes seemed to hold the key, so I thought about the pattern in those terms:

Term numbers The pattern is:	Growth Cycle Term	Pebble numbers $P_1 = 8$, so start with 8	
$T_1, T_2, T_3\ T_4$	1	$P_2 = 12$ $P_3 = 18$ $P_4 = 27$	$\frac{3 \times P_{n-1}}{2}$
$T_4, T_5, T_6\ T_7$	2	$P_5 = 45$ $P_6 = 75$ $P_7 = 125$	$\frac{5 \times P_{n-1}}{3}$
$T_7, T_8, T_9\ T_{10}$	3	$P_8 = 225$ $P_9 = 405$ $P_{10} = 729$	$\frac{9 \times P_{n-1}}{5}$

I noticed that the cube numbers which appear in the pattern, are repeated in the expressions, e.g.

$T_1, T_2, T_3\ T_4$ starts with $T_1 = 2^3$ and ends with $T_4 = 3^3$, these repeat : $\frac{3 \times P_{n-1}}{2}$

which continues, $T_4 = 3^3$ and ends with $T_7 = 5^3$, these repeat : $\frac{5 \times P_{n-1}}{3}$ etc...

Based upon the differentials between the numbers 3, 5, 9, I could predict the numerator to be: 3, 5, 9, 17, 33... (because the differentials double: 2, 4, 8, 16...) This means that the line of difference can be described as $2^1, 2^2, 2^3, 2^4$ ...

I could now write a general rule for the numerator as $2^n + 1$

Similarly I could see a doubling pattern in the denominators, so predicted 2, 3, 5, 9, 17, etc... This time the differentials are 1, 2, 4, 8, etc... so the line of difference can be described as $2^0, 2^1, 2^2, 2^3$ ...

Again, I could write a general rule for the denominator as $2^{n-1} + 1$

Finally I could write an expression to predict the ratios for growth cycles

$$\frac{(2^n + 1)\,P_{n-1}}{2^{n-1} + 1}$$

Remember that the string $T_{10,}\, T_{11,}\, T_{12}\ T_{13}$ is actually the FOURTH TERM in the growth cycle: GC_4

e.g. GC_4

$$\frac{(2^4 + 1)P_{n-1}}{2^3 + 1} = \frac{17 \times P_{n-1}}{9}$$

So for example, $P_{11} = \frac{17 \times 729}{9} = 1377$

Because the pebble numbers are recursive, you still need to know that $P_1 = 8$, and work through the sequence, but at least now I can predict pebble numbers.

Volume just doubles, so we are looking at a geometric sequence: 1, 2, 4, 8, 16, 32, 64, 128, etc...
where $V_1 = 1 \times 2^0$, $V_2 = 1 \times 2^1$, $V_3 = 1 \times 2^2$ ...
The exponent is always one less than the term number, so the rule is $V_n = 2^{(n-1)}$ where V = volume.

Surface area was 6, 10, 16, 24, 40, 64, 96, 160, etc...
The easiest pattern to spot was in the cubes, which are: 6, 24, 96, 384...
and can be described as 6×4^0, 6×4^1, 6×4^2 ... leading to a general rule for cubes only as: $6 \times 4^{n-1}$

There isn't a linear pattern in the surface area sequence, it's cyclical, like the pebble numbers. The numbers increased by 4, 6, 8, 16, 24, 32, etc... This meant that I could write a rule for each growth cycle:

Term numbers	Growth Cycle Term	Surface Area numbers		
$T_1, T_2, T_3\ T_4$	1	$SA_1 = 6$ $SA_2 = 10$ $SA_3 = 16$ $SA_4 = 24$	$SA_1 = 1^2 + 1 + 4 = 6$ $SA_2 = 2^2 + 2 + 4 = 10$ $SA_3 = 3^2 + 3 + 4 = 16$ $SA_4 = 4^2 + 4 + 4 = 24$	$SAGC_1 = n^2 + n + 4$
$T_4, T_5, T_6\ T_7$	2	$SA_4 = 24$ $SA_5 = 40$ $SA_6 = 64$ $SA_7 = 96$	$SA_1 = 4(1^2 + 1 + 4) = 24$ $SA_2 = 4(2^2 + 2 + 4) = 40$ $SA_3 = 4(3^2 + 3 + 4) = 64$ $SA_4 = 4(4^2 + 4 + 4) = 96$	$SAGC_2 = 4n^2 + 4n + 16$, or $4(n^2 + n + 4)$
$T_7, T_8, T_9\ T_{10}$	3	$SA_7 = 96$ $SA_8 = 160$ $SA_9 = 256$ $SA_{10} = 384$	$SA_1 = 4^2(1^2 + 1 + 4) = 96$ $SA_2 = 4^2(2^2 + 2 + 4) = 160$ $SA_3 = 4^2(3^2 + 3 + 4) = 256$ $SA_4 = 4^2(4^2 + 4 + 4) = 384$	$SAGC_3 = 16n^2 + 16n + 64$, or $16(n^2 + n + 4)$, or

I realised that I was using exponents of 4.

This meant that the expressions should read: $\mathbf{4^0(n^2+n+4),\ 4^1(n^2+n+4),\ 4^2(n^2+n+4),}$ **etc..** They themselves form a pattern, which I suspect will continue and describe the cube, cuboid, larger cuboid, cube growth pattern.

So if you think of the cycles as terms, the surface area of growth cycle 1 is $4^0(n^2+n+4)$, the surface area of growth cycle 2 is $4^1(n^2+n+4)$, the surface area of growth cycle 3 is $4^2(n^2+n+4)$, etc... This finally meant that I could now write a general rule:

The surface area of growth cycles (cube, cuboid, larger cuboid, cube) is $SAGC_n = 4^{(n-1)}(n^2+n+4)$

e.g. $SA_{11} = SA_2$ in $SAGC_4 = 4^{(4-1)}(2^2+2+4) = 640$

Again, it's a bit complicated because of the cyclical pattern, but at least I can predict surface area numbers.

External pebbles are:	8	12	18	26	42	66	98	162	258	386
First line of difference	4	6	8	16	24	32	64	96	128	
Second line of difference		2	2	8	8	8	32	32	32	

This isn't linear, but can be described in a similar way to the surface area. This time the expressions read:

Growth cycle 1 = $n^2+n+6 \rightarrow 4^0(n^2+n)+(6\times 1)$
Growth cycle 2 = $4n^2+4n+18 \rightarrow 4^1(n^2+n)+(6\times 3)$
Growth cycle 3 = $16n^2+16n+66 \rightarrow 4^2(n^2+n)+(6\times 11)$

Although the last number is divisible by 6, I wasn't able to write a general rule for the growth cycle of external pebbles (sorry).

Internal pebbles are a cube/ cuboid within a cube/cuboid. The sequence is 0, 0, 0, 1, 3, 9, 27, 63, 147, 343 etc... Initially I thought that the sequence increased threefold, but this broke down with the 9th term.

But, if you only look at the cubes, 0, 1, 27, 343... as $0^3, 1^3, 3^3, 7^3$ ... the differential between the numbers double (1, 2, 4... and I predict 8), so I predict that the internal pebble number of the next cube will be 15^3 or 3375.

There is probably a calculation you could do to formulate external + internal = whole, and if I get chance I'll try come back to this one, because I would like to know how this all fits together.