Inner Equality
Nrich Short Problem - Adithya Venkat (14)

The four numbers a,b,c,d lie between −5 and 5. In addition to this , the numbers are constrained further so that $5 < a + b < 10$ and $-10 < c + d < -5$. Let's also suppose that $0 < a , b < 5$ and $-5 < c , d < 0$.

1) ?? < a + b − c−d < ? ?

Firstly , I attempted to calculate the minimum and maximum values for -c-d using the inequality $-10 < c + d < -5$. Subtracting -d from -c can be a maximum of 10 if both c and d are equal to five: -(5)-(-5)=10. If c or d cannot equal zero at the same time , then the minimum value that -c-d has to be greater than is 5 , leaving the inequality $5 < - c - d < 10$. Due to the fact that $5 < a + b < 10$, we can add both minimum and maximum values of the two inequalities to give **$10 < a + b - c - d < 20$**.

2) ?? < a − c < ??

As previously established , $0 < a < 5$ and $0 < - c < 5$ therefore subtracting c , which lies between 0 and -5 , from a can produce a minimum total of 0 , if both a and c are equal to 0, and a maximum of 10 , occurring when both a and - c are equal to 5. Thus, the inequality representing the subtraction of c from a is **$0 < a - c < 10$**.

3) ?? < a − c + d − b < ??

Since I have worked out the range of values for where a - c lies, I can substitute this inequality into the centre to give ?? < (0 < a - c < 10) + d - b < ?? . On the condition that we take the value b to equal a number between 0 and 5 whilst on the other hand, the value d to equal a number between -5 and 0, because the in this instance I'm dealing with a negative b the minimum value for this particular subtraction would be -10 , occurring when b = 5 and d = -5 as -5 - + 5 = -10. This leaves just the maximum value to be calculated and this can be achieved through finding the two greatest numbers d and b can be equivalent to. Both negative b and d lie in between -5 and 0 therefore the maximum value is 0+0= 0, giving $-10 < d - b < 0$. Combining the inequalities expressing the range of values where a - c and d - b lie gives **$-10 < a - c + d - b < 10$** because the sum of the minimum values is -10 + 0= -10 and the sum of the maximum values is 0 + 10 = 10.

4) ?? < *abcd* < ??

Using the inequalities $0 < a , b < 5$ and $-5 < c , d < 0$, I figured out that the maximum value for the product will be 5 * 5 * -5 * -5 = 25^2 = 625. The minimum value for the product of abcd will be 0 as if either one of the numbers equals 0 , the product will be 0 due to the fact that any number multiplied by zero gives 0. **$0 < abcd < 625$**