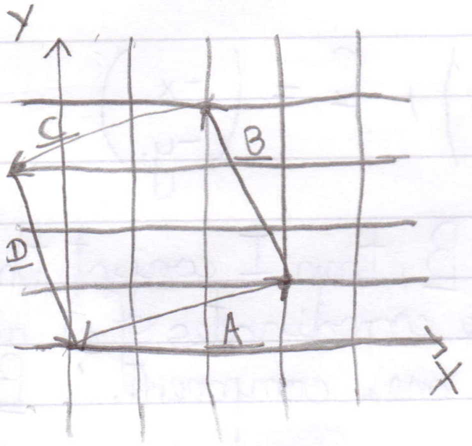
8th March 2011 Vector Journeys Niharika Paul



$\underline{A} = \begin{pmatrix}3\\1\end{pmatrix}$

$\underline{B} = \begin{pmatrix}-1\\3\end{pmatrix}$

$\underline{C} = -\underline{A} = \begin{pmatrix}-3\\-1\end{pmatrix}$

$\underline{D} = -\underline{B} = \begin{pmatrix}1\\-3\end{pmatrix}$

Another route:

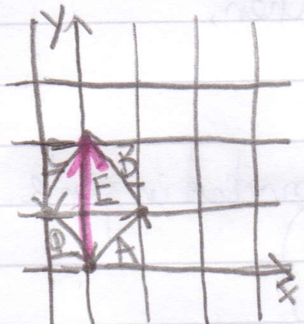


Fig. 1

He starts his journey by walking along $\underline{A} = \begin{pmatrix}1\\1\end{pmatrix}$

$\underline{B} = \begin{pmatrix}-1\\1\end{pmatrix}$, $\underline{C} = -\underline{A} = \begin{pmatrix}-1\\-1\end{pmatrix}$, $\underline{D} = \begin{pmatrix}1\\-1\end{pmatrix}$

Another route: $\underline{A} = \begin{pmatrix}1\\0\end{pmatrix}$, $\underline{B} = \begin{pmatrix}0\\1\end{pmatrix}$, $\underline{C} = \begin{pmatrix}-1\\0\end{pmatrix}$, $\underline{D} = \begin{pmatrix}0\\-1\end{pmatrix}$

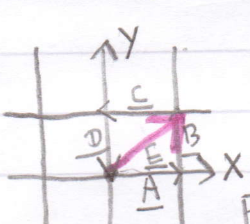


Fig. 2

Once you know $\underline{A}$ you can determine $\underline{B}$, $\underline{C}$, $\underline{D}$. $\underline{A}$ has an opposite direction to $\underline{C}$ ~~vector~~ and $|\underline{A}| = |\underline{C}|$.

$\underline{B}$ is 90° to $\underline{A}$ and $|\underline{B}| = |\underline{A}|$. So we know $\underline{B}$.

$\underline{B}$ and $\underline{D}$ have opposite directions and $|\underline{B}| = |\underline{D}|$. So we ~~know~~ know $\underline{D}$

I have noticed the following pattern ~~betw~~ amongst $\underline{A}$, $\underline{B}$, $\underline{C}$, $\underline{D}$

If $\underline{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\underline{C} = \begin{pmatrix} -x_1 \\ -y_1 \end{pmatrix}$

Also, if $\underline{A} \perp \underline{B}$, then I conject that to get $\underline{B}$ from $\underline{A}$ you swap the coordinates of $\underline{A}$ around and change the sign of one component. $\therefore$ $\underline{B} = \begin{pmatrix} -y_1 \\ x_1 \end{pmatrix}$ or $\begin{pmatrix} y_1 \\ -x_1 \end{pmatrix}$.

$\therefore$ $\underline{D} = \begin{pmatrix} y_1 \\ -x_1 \end{pmatrix}$ or $\begin{pmatrix} -y_1 \\ x_1 \end{pmatrix}$ respectively

This pattern $\Rightarrow$ if $\underline{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ then,

$(x_1 x_2) + (y_1 y_2) = 0$

In Fig. 1 and Fig. 2 Alison's path is marked in pink.

In Fig. 1 $\underline{E} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. In Fig 2! $\underline{E} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

We realise $\underline{E} = \underline{A} + \underline{B}$

Using Pythagoras

$|\underline{E}|^2 = |\underline{A}|^2 + |\underline{B}|^2$

If Charlie's path is a square,

$|\underline{E}|^2 = 2|\underline{A}|^2$

and $\underline{E}$ is 45° to $\underline{A}$

$\therefore$ We can know $\underline{E}$ given $\underline{A}$

Here's How: $\underline{A} + \underline{B} = \underline{E}$

$\begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\therefore x_1 + x_2 = 2$ —— 1

$y_1 + y_2 = 4$ —— 2

~~$\therefore \underline{B} = \begin{pmatrix} -y_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ -y_1 \end{pmatrix}$~~ $\therefore \underline{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} -y_1 \\ x_1 \end{pmatrix}$ or $\begin{pmatrix} y_1 \\ -x_1 \end{pmatrix}$

In the first case:
or $-y_1 = x_2$ and $y_2 = x_1$

$x_1 - y_1 = 2$ — ③
$y_1 + x_1 = 4$ — ④

$\therefore$
$$\begin{array}{r} x_1 - y_1 = 2 \\ + \; x_1 + y_1 = 4 \\ \hline 2x_1 \quad\; 6 \\ \hline \end{array}$$

$\therefore x_1 = 3$
$y_2 = 3$
$x_2 = -1$
$y_1 = 1$
$\underline{A} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \underline{B} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

In the 2^{nd} case:

$x_1 + y_1 = 2$ — ③
$y_1 - x_1 = 4$ — ④

$\therefore y_1 = 3$
$x_1 = -1$
$x_2 = 3$
$y_2 = 1$
$\underline{A} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $\underline{B} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

Alison's journey can be uniquely defined by a given $\underline{A}$ on the grid. You measure $45°$ and draw a line $\sqrt{2} \times \underline{A}$.